EC114 - INTRODUCTION TO QUANTITATIVE ECONOMICS

TEST: DRAFT Solutions

Question 1

1. *[5 marks]* Non-mutually exclusive events: $\Pr(DC \text{ or } MP) = \Pr(DC)+\Pr(MP)-\Pr(DC \text{ and } MP) = \frac{80}{300} + \frac{108}{300} - \frac{38}{300} = \frac{150}{300} = 50\%$

2. *[5 marks]* $\Pr(MP|DC) = \frac{\Pr(MP \text{ and } DC)}{\Pr(DC)} = \frac{38/300}{80/300} = \frac{38}{80} = 0.475$

3. *[5 marks]* We have: $\Pr(S) = \frac{240}{300} = \frac{4}{5}$; $\Pr(DC) = \frac{80}{300} = \frac{4}{15}$; $\Pr(S \text{ and } DC) = \frac{64}{300} = \frac{16}{75}$.
   Since $\Pr(S) \times \Pr(DC) = \frac{4}{5} \times \frac{4}{15} = \frac{16}{75} = \frac{64}{300} = \Pr(S \text{ and } DC)$, these events are independent. The independence can also be seen from the conditional probability:
   $\Pr(S|DC) = \frac{\Pr(S \text{ and } DC)}{\Pr(DC)} = \frac{64/300}{80/300} = \frac{64}{80} = 0.80 = \frac{240}{300} = \Pr(S)$

4. *[10 marks]* $\Pr(S|F) = \frac{\Pr(S \text{ and } F)}{\Pr(F)} = \frac{\Pr(F|S)\times\Pr(S)}{\Pr(F|S)\times\Pr(S)+\Pr(F|NS)\times\Pr(NS)} = \frac{0.80\times0.40}{0.80\times0.40+0.30\times0.60} = \frac{0.32}{0.32+0.18} = \frac{0.32}{0.50} = 0.64$

Question 2

1. *[6 marks]* $\Pr(Z > -0.93) = 0.824$

2. *[9 marks]* $\Pr(30 < X < 490) = \Pr\left(\frac{30-490}{90} < Z < \frac{490-490}{90}\right) = \Pr(5.11 < Z < 0) = 0.50$

3. *[10 marks]* We require k such that $\Pr(X > k) = 0.05$. From the table we find $\Pr(Z > 1.645) = 0.05$, so $\frac{k-4000}{60} = 1.645$. So: $k = 4,000 + 60(1.645) = 4,098.7$

Question 3

1. *[4 marks]* $E(X) = \sum xp(x) = 200$

2. *[4 marks]* $\sigma_X^2 = E[(X)^2]-[E(X)]^2 = 129,000-40,000 = 89,000$, so $\sigma_X = 298.33$

3. *[4 marks]* $E(\pi) = -5,000 + 35\mu_X = 2,000$

4. *[3 marks]* $\sigma_\pi = 35\sigma_X = 35 \times 298.33 = 10,441.55$

5. *[10 marks]* $\Pr(X < 2) = \Pr(X = 0) + \Pr(X = 1) = \binom{5}{0} \times 0.60^0 \times (0.40)^5 + \binom{5}{1} \times 0.60^1 \times (0.40)^4 = 0.08704$

Question 4

1. *[5 marks]*

	Z		
W	3	5	$p(w)$
0	0.12	0.18	0.30
1	0.08	0.12	0.20
3	0.20	0.30	0.50
$p(z)$	0.40	0.60	1

2. *[5 marks]* $\Pr(W + Z > 5) = 0.12 + 0.20 + 0.30 = 0.62$

3. *[5 marks]* $p(0.3) = 0.12 = 0.30 \times 0.40 = p(0) \times p(3)$, you check this is true for all the cells, so they are independent.

4. *[5 marks]* $\mu_W = 20\mu_X + 30\mu_Y = 1,700$; $\sigma_W^2 = 20^2\sigma_X^2 + 30^2\sigma_Y^2 + 2 \times 20 \times 30\rho_{X,Y}\sigma_X\sigma_Y = 93,780$

5. *[5 marks]* $Z_W = \frac{2,000-1,700}{306.24} = 0.980$; $\Pr(W > 2,000) = \Pr(Z_W > 0.980) = 0.164$