

EC114 Introduction to Quantitative Economics
Autumn Mid-Term Test (Resit) Solutions

1. The following table contains the relevant calculations (the sample size $n = 9$):

	X	$X - \bar{X}$	$(X - \bar{X})^2$	X^2
	5	-9	81	25
	6	-8	64	36
	10	-4	16	100
	10	-4	16	100
	15	1	1	225
	15	1	1	225
	15	1	1	225
	23	9	81	529
	27	13	169	729
Sums	126	0	430	2194

(a) [15 marks] The sample *mean*, \bar{X} , is defined by

$$\bar{X} = \frac{\sum_i X_i}{n} = \frac{126}{9} = 14.$$

The sample *mode* is the most common, or most frequent, value. Here, the mode is equal to 15 as this value occurs three times.

The sample *median* is the value in the middle of the (ordered) observations. Here, $n = 9$ so we want the value that has four below it and four above it i.e. the fifth value; hence the median is 15.

(b) [10 marks] The sample *variance* is defined by

$$s^2 = \frac{\sum_i (X_i - \bar{X})^2}{n - 1} = \frac{430}{8} = 53.75.$$

An alternative expression is

$$s^2 = \frac{\sum_i X_i^2}{n - 1} - \frac{n}{n - 1} \bar{X}^2 = \frac{2194}{8} - \frac{9}{8} 14^2 = 274.25 - 220.5 = 53.75.$$

The sample *standard deviation* is the (positive) square root of s^2 , s . Here we have $s = \sqrt{53.75} = 7.33$.

(c) [15 marks] Assuming that $X \sim N(\mu, 16)$ a 90% confidence interval for μ is of the form

$$\bar{X} \pm k \frac{\sigma}{\sqrt{n}},$$

where k is the value from the standard normal distribution that puts 5% into each tail. From the table we find that $k = 1.645$; we know that $\sigma^2 = 16$ and $n = 9$. Hence the 90% confidence interval is

$$\bar{X} \pm 1.645 \frac{4}{\sqrt{9}} = \bar{X} \pm 2.19.$$

We are therefore 90% confident that μ lies in the interval 11.81 to 16.19.

2. Let π denote the probability of having had higher education; here $\pi = 0.25$. The number of 'trials' (members of the randomly-chosen group) is $n = 8$. The binomial formula for the probability of X successes is

$$p(X) = \frac{n!}{(n-X)!X!} \pi^X (1-\pi)^{n-X}.$$

- (a) [10 marks] The probability of no individual in the group having had higher education is equal to the probability of zero successes. We therefore need to calculate

$$p(0) = \frac{8!}{8!0!} 0.25^0 (1-0.25)^8 = 0.75^8 = 0.10.$$

- (b) [10 marks] Here we require $p(4)$:

$$p(4) = \frac{8!}{(8-4)!4!} 0.25^4 (1-0.25)^{8-4} = 70 \times 0.0039 \times 0.3164 = 0.0864.$$

3. We are told that $X \sim N(5, 4)$.

- (a) [15 marks] We need $\Pr(X < 0)$. Using standardisation to transform to a $N(0, 1)$ random variable we obtain:

$$\begin{aligned} \Pr(X < 0) &= \Pr\left(\frac{X - \mu}{\sigma} < \frac{0 - 5}{2}\right) \\ &= \Pr(Z < -2.5) = \Pr(Z > 2.5) \\ &= 0.5 - \Pr(0 < Z < 2.5) \\ &= 0.5 - 0.4938 = 0.0062. \end{aligned}$$

- (b) [15 marks] We need to find $\Pr(2 < X < 8)$. Using standardisation to transform to a $N(0, 1)$ random variable we obtain:

$$\begin{aligned} \Pr(2 < X < 8) &= \Pr\left(\frac{2-5}{2} < \frac{X-\mu}{\sigma} < \frac{8-5}{2}\right) \\ &= \Pr(-1.5 < Z < 1.5) \\ &= 2 \Pr(0 < Z < 1.5) \\ &= 2 \times 0.4332 = 0.8664. \end{aligned}$$

- (c) [10 marks] To obtain a return of £4 on an investment of £100 implies a rate of return of 4%. We therefore need $\Pr(X > 4)$. Standardising we obtain:

$$\begin{aligned} \Pr(X > 4) &= \Pr\left(\frac{X - \mu}{\sigma} > \frac{4 - 5}{2}\right) \\ &= \Pr(Z > -0.5) = \Pr(Z < 0.5) \\ &= 0.5 + \Pr(0 < Z < 0.5) = 0.5 + 0.1915 = 0.6915. \end{aligned}$$