University of Essex
Session 2010–11

Department of Economics
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EC114 Introduction to Quantitative Economics

Autumn Mid-Term Test Solutions

1. The following table contains the relevant calculations (the sample size $n = 9$):

	X	$X - \bar{X}$	$(X - \bar{X})^2$	X^2
	5	−13	169	25
	10	−8	64	100
	12	−6	36	144
	15	−3	9	225
	17	−1	1	289
	20	2	4	400
	20	2	4	400
	25	7	49	625
	38	20	400	1444
Sums	162	0	736	3652

(a) [15 marks] The sample *mean*, $\bar{X}$, is defined by

$$\bar{X} = \frac{\sum_i X_i}{n} = \frac{162}{9} = 18.$$

The sample *mode* is the most common, or most frequent, value. Here, the mode is equal to 20 as this is the only value that occurs more than once. The sample *median* is the value in the middle of the (ordered) observations. Here, $n = 9$ so we want the value that has four below it and four above it i.e. the fifth value; hence the median is 17.

(b) [10 marks] The sample *variance* is defined by

$$s^2 = \frac{\sum_i (X_i - \bar{X})^2}{n-1} = \frac{736}{8} = 92.$$

An alternative expression is

$$s^2 = \frac{\sum_i X_i^2}{n-1} - \frac{n}{n-1}\bar{X}^2 = \frac{3652}{8} - \frac{9}{8}18^2 = 456.5 - 364.5 = 92.$$

The sample *standard deviation* is the (positive) square root of s^2, s. Here we have $s = \sqrt{92} = 9.59$.

(c) [15 marks] Assuming that $X \sim N(\mu, 9)$ a 95% confidence interval for μ is of the form

$$\bar{X} \pm k\frac{\sigma}{\sqrt{n}},$$

where k is the value from the standard normal distribution that puts 2.5% into each tail. From the table we find that $k = 1.96$; we know that $\sigma^2 = 9$ and $n = 9$. Hence the 95% confidence interval is

$$\bar{X} \pm 1.96\frac{3}{\sqrt{9}} = \bar{X} \pm 1.96.$$

We are therefore 95% confident that μ lies in the interval 16.04 to 19.96.

2. Let π denote the probability of success (i.e. of getting the question correct); here $\pi = 0.25$. The number of trials is $n = 10$. The binomial formula for the probability of X successes is

$$p(X) = \frac{n!}{(n-X)!X!}\pi^X(1-\pi)^{n-X}.$$

   (a) [10 marks] The probability of getting all ten questions wrong is equal to the probability of zero successes. We therefore need to calculate

$$p(0) = \frac{10!}{10!0!}0.25^0(1-0.25)^{10} = 0.75^{10} = 0.0563.$$

   (b) [10 marks] Here we require $p(4)$:

$$p(4) = \frac{10!}{(10-4)!4!}0.25^4(1-0.25)^{10-4} = 210 \times 0.0039 \times 0.1780 = 0.1458.$$

3. We are told that $X \sim N(3000, 800^2)$.

   (a) [15 marks] We need $\Pr(X > 4600)$. Using standardisation to transform to a $N(0,1)$ random variable we obtain:

$$\begin{aligned}\Pr(X > 4600) &= \Pr\left(\frac{X-\mu}{\sigma} > \frac{4600-3000}{800}\right)\\ &= \Pr(Z > 2)\\ &= 0.5 - \Pr(0 < Z < 2)\\ &= 0.5 - 0.4772 = 0.0228.\end{aligned}$$

   (b) [15 marks] We need to find $\Pr(2500 < X < 3500)$. Using standardisation to transform to a $N(0,1)$ random variable we obtain:

$$\begin{aligned}\Pr(2500 < X < 3500) &= \Pr\left(\frac{2500-3000}{800} < \frac{X-\mu}{\sigma} < \frac{3500-3000}{800}\right)\\ &= \Pr(-0.625 < Z < 0.625)\\ &= 2\Pr(0 < Z < 0.625)\\ &= 2 \times 0.234 = 0.468.\end{aligned}$$

   Note the value of $\Pr(0 < Z < 0.625) = 0.234$ has been obtained by taking the midpoint between the two probabilities for $\Pr(0 < Z < 0.62) = 0.2324$ and $\Pr(0 < Z < 0.63) = 0.2357$; anything in this range is acceptable.

   (c) [10 marks] We need to find M such that $\Pr(-\infty < X < M) = 0.95$. Standardising we obtain:

$$\begin{aligned}\Pr(-\infty < X < M) &= \Pr\left(\frac{\infty-3000}{800} < \frac{X-\mu}{\sigma} < \frac{M-3000}{800}\right)\\ &= \Pr\left(-\infty < Z < \frac{M-3000}{800}\right).\end{aligned}$$

   From the $N(0,1)$ table we need to find k such that $\Pr(-\infty < Z < k) = 0.95$; this is the value for which the table gives $\Pr(0 < Z < k) = 0.45$ which is $k = 1.645$. Hence we need to solve the following equation for M:

$$\frac{M-3000}{800} = 1.645 \quad \Rightarrow \quad M = 3000 + 1.645(800) = 4316.$$

   Hence the individual needs to insure losses up to the value of £4316.