University of Essex — Department of Economics
Session 2010–11 — Professor M. J. Chambers

EC114 Introduction to Quantitative Economics

Spring Mid-Term Test (Resit) Solutions

1. [20 marks] The null hypothesis is $H_0 : \mu = 95$ and the alternative is $H_A : \mu < 95$, where μ denotes the population mean. We have a large enough sample to use the central limit theorem result:

$$TS = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim N(0, 1)$$

under H_0, where $\bar{X}$ denotes the sample mean, s denotes the sample standard deviation, and n is the sample size. The 5% critical value from the $N(0, 1)$ distribution for a lower one-tail test is -1.645, so the decision rule is:

If $TS < -1.645$, reject H_0 in favour of H_A; do not reject otherwise.

We find that

$$TS = \frac{94.5 - 95}{2/\sqrt{90}} = \frac{-0.5}{0.2108} = -2.3719.$$

Hence $TS = -2.3719 < -1.645$ and so we reject H_0 in favour of H_A i.e. there is evidence to suggest that the railway company is failing to meet its target.

2. (a) [10 marks] We need to assume that we have a random sample and that sales are normally distributed. Due to the small sample and unknown variance we will have to rely on the t-distribution, hence the need for these assumptions.

   (b) [15 marks] We need to test $H_0 : \mu = 25$ against $H_A : \mu > 25$, where μ denotes the population mean. The test statistic is

$$TS = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$$

under H_0, where $\bar{X}$ denotes the sample mean, s denotes the sample standard deviation, and n is the sample size. The (upper) one-tailed 5% critical value from the t_3 distribution is 2.353. The decision rule is:

If $TS > 2.353$, reject H_0 in favour of H_A; do not reject otherwise.

We find that

$$TS = \frac{26 - 25}{\sqrt{9}/\sqrt{4}} = \frac{1}{3/2} = \frac{2}{3} = 0.67.$$

Hence $TS = 0.67 < 2.353$ and so we do not reject H_0 i.e. there is insufficient evidence to reject the null hypothesis that mean sales have remained unchanged.

(c) [15 marks] We wish to test $H_0 : \sigma^2 = 5$ against $H_A : \sigma^2 > 5$, where σ^2 denotes the population variance. The test statistic is

$$TS = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

under H_0. The 5% (upper) one-tail critical value from the χ^2_3 distribution is 7.815, and so the decision rule is:

If $TS > 7.815$, reject H_0 in favour of H_A; do not reject otherwise.

We find that

$$TS = \frac{3 \times 9}{5} = 5.4.$$

Hence $TS = 5.4 < 7.815$ and so we do not reject H_0 i.e. there is insufficient evidence to suggest that the variance of sales has risen, so we reserve judgment.

3. (a) [10 marks] The sample correlation coefficient is

$$R = \frac{\sum x_i y_i}{\sqrt{\sum y_i^2}\sqrt{\sum x_i^2}} = \frac{20.10}{\sqrt{25.64}\sqrt{20.46}} = \frac{20.10}{22.90} = 0.8777.$$

This suggests a strong positive linear association between monthly expenditure and income.

(b) [15 marks] The test statistic is

$$TS = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}} \sim t_{n-2}$$

under H_0. Here $n = 20$ and the 5% (upper) one-tail critical value from the t_{18} distribution is 1.734. The decision rule is:

If $TS > 1.734$, reject H_0 in favour of H_A; do not reject otherwise.

We find that

$$TS = \frac{0.8777\sqrt{18}}{\sqrt{1-0.8777^2}} = \frac{3.7238}{\sqrt{0.2296}} = 7.7714.$$

Hence $TS > 1.734$ and so we reject H_0 in favour of H_A i.e. there is evidence that the population correlation coefficient is positive.

(c) [15 marks] Plugging in the numbers we find that

$$b = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{20.10}{20.46} = 0.9824,$$

$$a = \bar{Y} - b\bar{X} = 5.55 - (0.9824 \times 5.59) = 0.0584.$$

The sample regression line is $Y = 0.0584 + 0.9824X + e$ where e denotes the residual, or $\hat{Y} = 0.0584 + 0.9824X$ where $\hat{Y}$ denotest the fitted value. The interpretation of b is that an additional hundred pounds of disposable income raises monthly expenditure by £98.24.