

UNIVERSITY OF ESSEX

SPRING MID-TERM TEST, THURSDAY 10 FEBRUARY 2011

INTRODUCTION TO QUANTITATIVE ECONOMICS

Time allowed: 60 minutes.

Write your name, your registration number, your class teacher's name, and your class number clearly on the front page of your answer book(s).

Candidates must answer ALL questions.

This paper consists of THREE questions.

Statistical tables can be found on pages 4, 5 and 6.

Candidates are allowed to bring into the examination room:
calculators (hand held, containing no textual information).

Please do not leave your seat unless you are given permission by an invigilator.

Do not communicate in any way with any other candidate in the examination room.

Do not open the question paper until told to do so.

All answers must be written in the answer book(s) provided.

All rough work must be written in the answer book(s) provided. A line should be drawn through any rough work to indicate to the examiner that it is not part of the work to be marked.

At the end of the examination, remain seated until your answer book(s) have been collected and you have been told you may leave.

1. [20 marks] A machine on a production line is designed to fill plastic bottles with 500ml of mineral water. To test whether the machine is working correctly a technician examines the contents of 100 bottles and finds that they have a mean of 499ml and a standard deviation of 10ml. Test whether the machine is working correctly, using a 5% level of significance, clearly specifying the null and alternative hypotheses being tested and the decision rule you employ.

2. Answer *all* parts (a) – (c) of this question.

A scientific study establishes that a number of health benefits can be obtained by eating raw tomatoes. A supermarket chain currently sells, on average, 10 tonnes of tomatoes per week with a variance of 4 tonnes, but will need to obtain more from its suppliers if consumers respond to the scientific study by wanting to buy more tomatoes. The supermarket chain monitors sales of tomatoes in the five weeks following publication of the study, finding a mean of 13 tonnes per week with a variance of 5 tonnes.

 - (a) [10 marks] What assumptions need to be made to test whether sales have increased based on this sample?
 - (b) [15 marks] Test whether sales have increased, using a 5% level of significance for your test.
 - (c) [15 marks] The supermarket chain also suspects that the variance of sales has increased. Test this hypothesis at the 5% level of significance.

Question 3 can be found on the next page.

3. Answer *all* parts (a) – (c) of this question.

In order to investigate the relationship between wages (Y), measured in £ per week, and education (X), measured in years, a sample of 25 individuals yields the following summary data:

$$\begin{aligned}\bar{Y} &= 140, & \sum y_i^2 &= 360,000, & \sum x_i y_i &= 25,000, \\ \bar{X} &= 8, & \sum x_i^2 &= 2,500,\end{aligned}$$

where \bar{Y} and \bar{X} denote the sample means of Y and X , respectively, $y_i = Y_i - \bar{Y}$ and $x_i = X_i - \bar{X}$.

(a) [10 marks] Calculate, and interpret, the sample correlation coefficient

$$R = \frac{\sum x_i y_i}{\sqrt{\sum y_i^2} \sqrt{\sum x_i^2}}.$$

(b) [15 marks] Using R , carry out a test of $H_0 : \rho = 0$ against $H_A : \rho > 0$ at the 5% level of significance, where ρ denotes the population correlation coefficient between Y and X . What do you conclude about the relationship between Y and X ?

(c) [15 marks] Assuming that $Y = \alpha + \beta X + \epsilon$, where ϵ is a random disturbance and α and β are unknown parameters, calculate the ordinary least squares (OLS) estimates of α and β using

$$b = \frac{\sum x_i y_i}{\sum x_i^2}, \quad a = \bar{Y} - b\bar{X}.$$

Write down the sample regression line and interpret the coefficient b .

(END OF PAPER)

Areas under the standard normal curve

z'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

NB: Entries are $\Pr(0 < Z < z')$ e.g. $\Pr(0 < Z < 1.32) = 0.4066$.

**Percentage points of Student's t -distribution with
 v degrees of freedom**

v	0.25	0.10	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
100	0.677	1.290	1.660	1.984	2.364	2.626
∞	0.674	1.282	1.645	1.960	2.326	2.576

Entries are t_α such that $\Pr(t_v > t_\alpha) = \alpha$, e.g. $\Pr(t_5 > 2.571) = 0.025$.

Percentage points of χ^2 -distribution with v degrees of freedom

v	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	0.0000393	0.000157	0.000982	0.00393	3.841	5.024	6.635	7.879
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597
3	0.0717	0.115	0.216	0.352	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Entries are c_α such that $\Pr(\chi_v^2 > c_\alpha) = \alpha$, e.g. $\Pr(\chi_6^2 > 14.449) = 0.025$.