University of Essex — Department of Economics
Session 2011–12 — Professor M. J. Chambers

EC114 Introduction to Quantitative Economics

Problem Set 14

Properties of Estimators

1. A variable X is normally distributed with mean μ and variance σ^2, and two researchers are given the task of estimating μ. A random sample of size $n = 2n_1$ is taken from the population, where n_1 is a positive integer. One researcher is given all even-numbered observations, and decides to use the estimator

$$\bar{X}_e = \frac{X_2 + X_4 + X_6 + \ldots + X_{n-2} + X_n}{n_1},$$

while the other is given all odd-numbered observations and decides to use

$$\bar{X}_o = \frac{X_1 + X_3 + X_5 + \ldots + X_{n-3} + X_{n-1}}{n_1}.$$

Note that both estimators are based on the same number of observations, n_1, although the observations determining each are entirely different.

   (a) Using Theorem 3.1 of Thomas (or otherwise), find the mean and variance of $\bar{X}_e$ and $\bar{X}_o$ and write down their distributions.

   (b) Find the mean square errors (MSEs) of the two estimators. Which estimator would you prefer to use, and why?

   (c) How do the properties of $\bar{X}_e$ and $\bar{X}_o$ compare with those of

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 + \ldots + X_{n-1} + X_n}{n},$$

   the full-sample mean? What does this imply about the use of sample information?

2. A variable X has mean μ and variance σ^2. Two independent random samples of observations on X of sizes n_1 and n_2 are taken with corresponding sample means $\bar{X}_1$ and $\bar{X}_2$ respectively. Two estimators of μ are proposed:

$$\hat{\mu} = \frac{\bar{X}_1 + \bar{X}_2}{2} \quad \text{and} \quad \tilde{\mu} = \left(\frac{n_1}{n}\right)\bar{X}_1 + \left(\frac{n_2}{n}\right)\bar{X}_2,$$

where $n = n_1 + n_2$.

   (a) Using Theorem 3.1 of Thomas (or otherwise) derive the bias, variance and mean square error (MSE) of both estimators.

   (b) Assuming that $n_1 = 20$ and $n_2 = 30$, which estimator would you prefer to use and why?

3. *(Optional Extension Question).* Show that, for an estimator Q of θ,

$$\text{MSE}(Q) = V(Q) + [\text{bias}(Q)]^2,$$

where $\text{MSE}(Q)$ denotes the mean square error of Q, $V(Q)$ is the variance of Q, and $\text{bias}(Q) = E(Q) - \theta$.