UNIVERSITY OF ESSEX

AUTUMN MID-TERM TEST, THURSDAY 24 NOVEMBER 2011

INTRODUCTION TO QUANTITATIVE ECONOMICS

ANSWERS

Question 1

The scores are:

$$10, 12, 7, 8, 14, 3, 15$$

a) The sample mean is defined as

$$\begin{aligned} \bar{X} &= \frac{\sum_{i=1}^{N} X_i}{N} \\ &= \frac{69}{7} \\ &= 9.86 \end{aligned}$$

to 2 decimal places.

The median is defined as X_4 i.e. the 4th value when the sample is arranged in increasing order. In this case the the scores in increasing order are $3, 7, 8, 10, 12, 14, 15$. so the median is 10.

b) We gave two defintions for the sample variance and standard deviation. First we gave:

$$\begin{aligned} v^2 &= \frac{\sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2}{N} \\ &= \frac{106.86}{7} \\ &= 15.27 \end{aligned}$$

to 2 decimal places.

The sample standard deviation is the square root of the sample variance:

$$\begin{aligned} v &= \sqrt{\frac{\sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2}{N}} \\ &= \sqrt{15.27} \\ &= 3.91 \end{aligned}$$

Alternative defintions are the 'corrected' sample variance and standard deviation:

$$\begin{aligned} s^2 &= \frac{\sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2}{N-1} \\ &= 17.81 \end{aligned}$$

and

$$\begin{aligned} s &= \sqrt{\frac{\sum_{i=1}^{N}\left(X_i-\bar{X}\right)^2}{N-1}} \\ &= 4.22 \end{aligned}$$

As further alternatives it is OK to formulate these expressions in terms of the sum of the squares. For example:

$$s^2 = \frac{\sum_{i=1}^{N} X_i^2}{N-1} - \frac{N}{N-1}\bar{X}^2$$

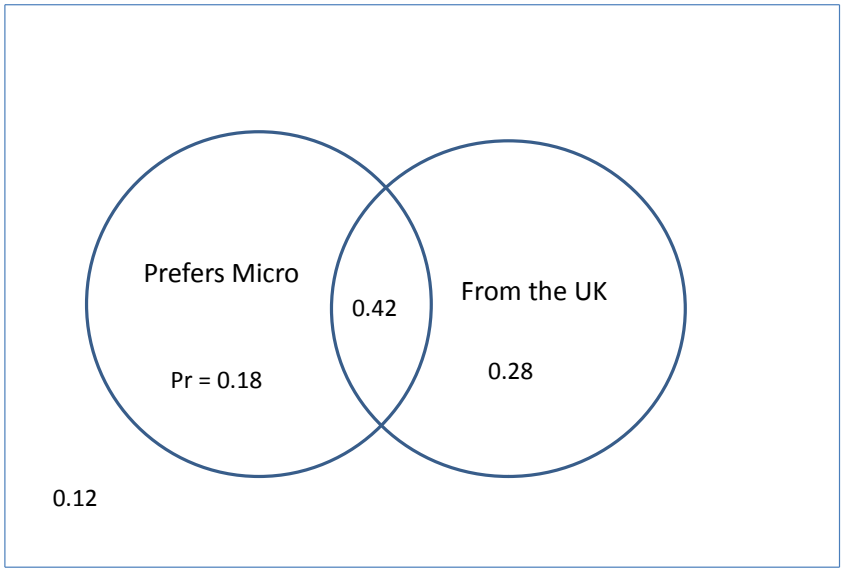
The mean absolute deviation is defined as $\frac{\sum_{i=1}^{N}|X_i-\bar{X}|}{N}$ (or $\frac{\sum_{i=1}^{N}|X_i-\bar{X}|}{N-1}$). Here this is 3.31 (or 3.86).

c) We know from lectures that v^2 is biased downwards ie. $\mathbb{E}\left(v^2\right) < \sigma^2$. But s^2, defined above, is an unbiased estimator: $\mathbb{E}\left(s^2\right) = \sigma^2$. As calculated above, $s^2 = 17.81$.

A sensible estimator of the population median is the sample median. As emphasized in lectures, the sample analogue of the population parameter is generally a 'good' estimator. For example the sample mean is an unbiased estimator of the population mean. Strictly speaking the sample median is only unbiased when the population distribution is symmetric, i.e. when the population mean is equal to the population median. This can be seen when we have a sample size of 1. Then $\mathbb{E}\left(\text{Med}\left(X\right)\right) = \mathbb{E}\left(X\right) = \mu$. In general $\mathbb{E}\left(\text{Med}\left(X\right)\right)$ will be between the population mean and the population median, but will converge to the population median as the sample size gets bigger. Full marks are given to any sensible discussion along these lines (in fact just for suggesting the sample median).

Question 2

a) A venn diagram of the event space with probabilities is:



b) Define the events: 'UK' = 'being from the UK'; '~UK'='coming from outside the UK'; 'macro' = 'preferring macro' and 'micro'='preferring micro'. Then

$$\begin{aligned} Pr\left(\text{UK } \textit{OR} \text{ micro}\right) &= Pr(\text{UK}) + Pr(\text{micro}) - Pr(\text{UK } \textit{AND} \text{ micro}) \\ &= 0.7 + 0.6 - 0.42 \\ &= 0.88 \end{aligned}$$

c) Yes these events are independent. We have that

$$\begin{aligned} Pr(\text{UK } \textit{AND} \text{ micro}) &= 0.42 \\ &= 0.7 \times 0.6 \\ &= Pr(\text{UK}) \times Pr(\text{micro}) \end{aligned}$$

This is one of the conditions for independence (and gets full marks).

Alternatively we could use $Pr(\text{UK } \textit{AND} \text{ micro}) = Pr\left(\text{UK}|\text{micro}\right) Pr\left(\text{micro}\right)$. This gives:

$$\begin{aligned} Pr\left(\text{UK}|\text{micro}\right) &= \frac{Pr(\text{UK } \textit{AND} \text{ micro})}{Pr\left(\text{micro}\right)} \\ &= \frac{0.42}{6} \\ &= 0.7 \\ &= Pr\left(\text{UK}\right) \end{aligned}$$

The condition $Pr\left(\text{UK}|\text{micro}\right) = Pr(\text{UK})$ is the first condition of independence defined in the lectures.

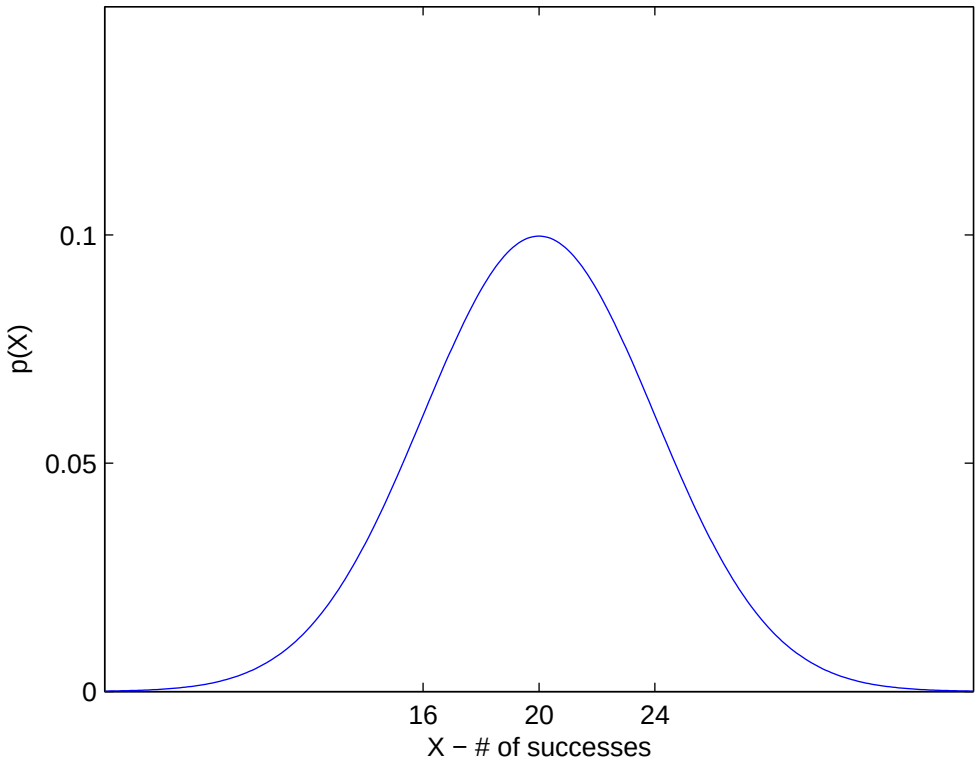
d) The Venn diagram shows that:

$$\begin{aligned} Pr(\text{macro}|\ \text{UK}) &= \frac{Pr(\text{macro } \textit{AND } \sim\text{UK})}{Pr(\ \text{UK})} \\ &= \frac{0.12}{0.3} \\ &= 0.4 \end{aligned}$$

Notice this is the same as the unconditional probability of preferring macro to micro. This is because if the events 'micro' and 'UK' are independent then so must be the events 'macro' and '~UK': a student's preference over subjects is independent of her country of origin.

Question 3

a) The (approximate) probability density function of the number of successes is:



b)

$$\begin{aligned} Pr\left(15 \leq X \leq 22\right) &= Pr\left(\frac{15-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{22-\mu}{\sigma}\right) \\ &= Pr\left(-\frac{5}{4} \leq Z \leq \frac{2}{4}\right) \\ &= Pr\left(0 \leq Z \leq \frac{5}{4}\right) + Pr\left(0 \leq Z \leq \frac{2}{4}\right) \end{aligned}$$

$$\begin{aligned} &= 0.3944 + 0.1915 \\ &= 0.5859 \end{aligned}$$

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c) The probability distribution of the number of successes can be modelled precisely as a binomial distribution. This is because the probability of winning is independent across time and presumably constant. Therefore we can calculate all the probabilities by noticing that the probability of success, π, $= 0.2$. Therefore the probability of zero successes is

$$\begin{aligned} (1-\pi)^{100} &= 0.8^{100} \\ &\approx 2.04 \times 10^{-10} \end{aligned}$$

a very small number!