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DEPARTMENT OF ECONOMICS
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EC 247 Financial Instruments and Capital Markets

Class Exercise 1

The questions are mainly chosen from chapter 3 of the Frederic S. Mishkin and Stanley G. Eakins, *Financial Markets and Institutions*, 7th Edition, Pearson Prentice Hall.

Chapter 3: This chapter introduces the concept of present value. We examine four basic types of instruments, which incorporate present value concepts: a) a simple loan, b) a fixed payment loan, c) a discount bond and d) a coupon bond. The chapter explains what the interest rate is, that it is negatively associated with the price of a bond, that it may differ from the return on a bond, and that there is an important distinction between real and nominal interest rates.

A fundamental principle of finance (and economics):
Any payment received at some point in the future is worth less than the same payment received today.

Present value: The value now of a sum due for payment at some point in the future.

$$PV = \frac{CF}{(1+i)^n}$$

Yield to maturity: The interest rate that equates the present value of cash flows received from a debt instrument with its value today; i.e. i in the PV formula above.

Fixed Payment Loan:

$$LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + ... + \frac{FP}{(1+i)^n}$$

Where:
LV = loan value
FP = Fixed yearly cash flow payment
n = number of years until maturity

Coupon Bond:

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + ... + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Where:
P = price of coupon bond
C = yearly coupon payment
F = face value of a bond
n = number of years until maturity

The yield to maturity for a one year pure discount (zero-coupon) bond:

$$i = \frac{F - P}{P}$$

Where:

P = current price of the coupon bond

F = face value of the bond

The yield to maturity, or spot yield, on an n-period Zero Coupon bond is defined as the constant annual rate of return i_n, that would be received if the bond is held until maturity:

$$i_n = \left(\frac{F}{P_n}\right)^{\frac{1}{n}} - 1$$

The spot yield is the rate of return on the ZC bond only if it is held to maturity. Notice that the yield to maturity on a coupon-paying bond does not have the same interpretation as the spot yield on a Zero-Coupon bond. More specifically, it need *not* be the case that the rate of return from holding a coupon-paying bond from the present until maturity equals the value of i that satisfies:

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

This is so, because the stream of payments received between the date of purchase and maturity may, of necessity, be reinvested at rates different from i. In fact, the coupons may not be reinvested at all. Only if every coupon is reinvested (from the date of its receipt until maturity) at rate i, will the rate of return from holding the bond until it matures equal the yield to maturity, i, as calculated from the present value formula above. As a result, for coupon-paying bonds the value of i is, at best, an approximation to the rate of return from holding the bond from the present until it matures.

Perpetuity or Consol: Fixed coupon payment of $\$C$ forever:

$$P_c = \frac{C}{i_c} \qquad\qquad i_c = \frac{C}{P_c}$$

Where:

P_c = price of the perpetuity (consol)

C = yearly payment

i_c = yield to maturity of the perpetuity (consol)

The above formula, which gives the yield to maturity for a perpetuity provides a useful approximation for the yield to maturity on coupon bonds with long term to maturity. Consequently, i_c, which is the yearly coupon payment divided by the price of the security is called 'current yield' and is used as an approximation to describe interest rates on long-term bonds.

Solutions

Question 1

a. Calculate the present value of $1,000 zero-coupon bond with 5 years to maturity if the required annual interest rate is 6%.
b. Repeat the calculation for an interest rate of 7%.
c. Calculate the present value if the interest rate is 6% with 6 years to maturity.

Solution:

a. $FV = \$1000,$

$i = 0.06,$

$n = 5$

$PV = ?$

$$PV = \frac{FV}{(1+i)^n} = \frac{\$1000}{(1+0.06)^5} = \$747.25$$

b. $FV = \$1000,$

$i = 0.07,$

$n = 5$

$PV = ?$

$$PV = \frac{FV}{(1+i)^n} = \frac{\$1000}{(1+0.07)^5} = \$712.98$$

For a given maturity, the bond's current price decreases as yield to maturity rises.

c. $FV = \$1000,$

$i = 0.06,$

$n = 6$

$PV = ?$

$$PV = \frac{FV}{(1+i)^n} = \frac{\$1000}{(1+0.06)^6} = \$704.96$$

For a given yield to maturity, the bond's price decreases as its maturity increases.

Question 2

Assume you just deposited $1,000 into a bank account. The current real interest rate is 2% and inflation is expected to be 6% over the next year. What nominal interest rate would you require from the bank over the next year? How much money will you have at the end of one year? If you are saving to buy a stereo that currently sells for $1,050, will you have enough to buy it?

This question emphasizes the distinction between nominal and real interest rates. Real interest rate is the rate that is adjusted by subtracting expected changes in the price level (inflation) so that to reflect more accurately the true cost of borrowing. This concept is extensively analyzed on pages (52-53/ 6th Edition) or (88-89 /7th Edition) in the main textbook.

Solution: The required nominal rate would be:

$$i = i_r + \pi^e$$
$$= 2\% + 6\% = 8\%.$$

At this rate, you would expect to have \$1,000 × 1.08, or \$1,080 at the end of the year.
Can you afford the stereo? In theory, the price of the stereo will increase with the rate of inflation. So, one year later, the stereo will cost \$1,050 × 1.06, or \$1,113. You will be short by \$33.

Question 3

A 10-year, 7% coupon bond with a face value of \$1,000 is currently selling for \$871.65. Compute your rate of return if you sell the bond next year for \$880.10.

Solution:

$$R = \frac{C + P_{t+1} - P_t}{P_t} = \frac{\$70 + \$880.10 - \$871.65}{\$871.65} = 0.09, \quad \text{or} \quad 9\%.$$

Question 4

Suppose that a corporate bond has a coupon rate of 20%, a face value of £1,000, a yield to maturity of 10%, and matures in two years. Obtain an expression for the price of the bond assuming semi-annual compounding.

The first step is to identify the cash flows that the holder of the bond will receive. These cash flows are then discounted back to the present in order to determine the price of the bond.

To adjust the cash flows to semi-annual payments we have to divide the coupon payment by 2 since only half of the annual payment is paid every six months. In the same notion, to find the interest rate effective during one-half of the year we have to divide the market interest rate by 2. Finally, we have to double the number of periods since there are two periods per year.

The appropriate formula for valuing semiannual bonds is the following:

$$P_{semi} = \frac{C/2}{1+i} + \frac{C/2}{(1+i)^2} + \frac{C/2}{(1+i)^3} + ... + \frac{C/2}{(1+i)^{2n}} + \frac{FV}{(1+i)^{2n}}$$

Where:
P_{semi} = price of semiannual coupon bond
C = yearly coupon payment

FV = face value of the bond
n = years to maturity
$i = \frac{1}{2}$ annual market interest rate

Using the above equation we can obtain an expression for the price of the bond in the question:

$$P_{semi} = \frac{£200/2}{(1+0.05)} + \frac{£200/2}{(1+0.05)^2} + \frac{£200/2}{(1+0.05)^3} + \frac{£200/2}{(1+0.05)^4} + \frac{£1000}{(1+0.05)^4}$$

We will discuss this issue in lecture 4 when we will analyse extensively the bond market. If you would like to read more at this stage please go to pages (252-254/ 6th Edition) or (335-338/ 7th Edition) in our textbook.

Question 5

What is the yield to maturity on a bond that has a price of £3,000 and pays £150 annually forever?

This is a perpetual bond with no maturity date and no repayment of principal that makes fixed coupon payments of £C forever. To calculate the yield to maturity for a perpetuity we use the following formula:

$$i_c = \frac{C}{P_c}$$

Where i_c = the yield to maturity for a perpetuity
C = the fixed coupon payment forever
P_c = the price of perpetuity (consol)

This formula provides a useful approximation for the yield to maturity on coupon bonds with a long term to maturity. Applying the numbers in the above formula we get:

$$i_c = \frac{C}{P_c} = \frac{£150}{£3{,}000} = 0.05 = 5\%$$

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