

Problem Set 1

1. Show that:

- a) $f(x_1, x_2) = x_1^{1/2} x_2^{1/3}$ is strictly concave
- b) $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ is (weakly) concave
- c) $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$ is (strictly) quasiconcave
- d) $f(x_1, x_2) = x_1^{1/3} x_2^{2/3}$ is (strictly) quasiconcave
- e) $f(x_1, x_2) = 3x_1^4 + 5x_2^2$ is (strictly) quasiconvex

for $x_1, x_2 > 0$. For questions (c)-(e) also check whether these functions are concave/convex.

2. A monopolist facing a demand function:

$$P = 100 - (Q_1 + Q_2)$$

produces two identical goods Q_1, Q_2 , with cost functions:

$$C_1 = 2Q_1^2$$

$$C_2 = 3Q_2^2.$$

Find the profit-maximizing levels of output. Is this a global maximum?

3. Suppose $f(x, y) = x^\alpha y^\beta$ with $x, y > 0$.

- a) Show that $f(x, y)$ is strictly concave if $\alpha, \beta > 0$ and $\alpha + \beta < 1$.
- b) Show that $f(x, y)$ is (strictly) quasiconcave for all $\alpha, \beta > 0$.

4. Find the maximum / minimum of

$$\Theta(x, y, z) = xz + x^2 - y + yz + y^2 + 3z^2$$