Problem Set 4

1. Solve the maximization problem

   $u = (x_1 + a)x_2^b$ s.t. $p_1x_1 + p_2x_2 \leq m$ and $x_1, x_2 \geq 0$

   where $a, b \geq 0$.

2. A central planner uses labour L to produce two outputs, x_1 and x_2, according to the production functions

   $$x_1 = 10L_1 - 0.5L_1^2$$
   $$x_2 = 8L_2 - 0.75L_2^2$$

   where L_i is labour allocated to sector i = 1,2. She wishes to maximise the value of total output

   $$V = 4x_1 + 5x_2$$

   where 4 and 5 are the world prices of x_1 and x_2 respectively.

   (a) Assuming there are 12 units of labour available, find the optimal labour allocation and the shadow price of labour.
   (b) Assume now that there are 20 units of labour available. What is the new optimal labour allocation and the shadow price of labour?

3. A consumer has the utility function

   $$u = x_1x_2$$

   and she faces a money- income constraint

   $$2x_1 + 3x_2 \leq 100$$

   and a time constraint

   $$x_1 + 4x_2 \leq 80.$$

   Solve for her utility–maximizing consumption bundle and the values of the shadow prices of the constraints.