Problem Set 5

1. A perfectly competitive industry has the supply function:

   $Q_t = F + Gp_t$ where F, G > 0 and t = 0, 1, 2, …

   The demand for this product is a function of price and the lagged value of quantity:

   $Q_t = A + Bp_t + \theta Q_{t-1}$ where $A > 0$, $B < 0$, $0 < \theta < 1$ and $t = 1, 2, 3, \ldots$

   Assuming that the market clears each period, derive a first-order difference equation for Q. Solve the difference equation and determine if the solution converges to the steady state.

2. Suppose that there are N identical pig farms. The total supply of pigs is given by:

   $$S = N(p_t - \alpha) / 2\beta$$

   where the price $p > \alpha$ and $\alpha, \beta > 0$.

   The demand for pigs is a function of next period's price:

   $$D = \theta - \delta\, p_{t+1}$$

   where θ and δ are positive constants. Assuming that the market clears each period, derive a first-order difference equation for p. Solve the difference equation, and determine if the solution converges to the steady state.

3. Consider the following model where price is no longer determined by a market - clearing mechanism but by the level of inventory $S_t - Q_t$:

   $$Q_t = c + zP_t$$
   $$S_t = g + hP_t$$
   $$P_{t+1} = P_t - a(S_t - Q_t)$$

   where a, h > 0 and z < 0. Thus a build-up in inventory ($S_t > Q_t$) will tend to reduce price and a depletion of inventory ($S_t < Q_t$) will cause price to rise. Find the price P_t for any period and comment on the stability conditions of the time path.

4. Consider the following lagged income determination model:

$$C_t = 200 + 0.75Y_{t-1}$$

$$I_t = 50 + 0.15Y_{t-1}$$

and $Y_0 = 3000$. Find the time path of national income Y and comment on the stability properties of the time path.