EC351 Mathematical Economics 2011/2012

Problem Set 6

1. For the following difference equations:
   a) $y_{t+1} = 3/16 + y^2_t$
   b) $y_{t+1} = 4 + (9 / 4y_t)$

   find the steady state points, determine whether they are locally stable and sketch a phase diagram to investigate global stability.

2. Consider the Solow growth model in discrete time. The per-capita production functions is: $y_t = k_t^{\alpha}$, where y_t is the per-capita output at time t, k_t is the per-capita capital level at time t and $0 < \alpha < 1$.
   The dynamics of capital accumulation is given by the following equation:
   $$k_{t+1} = k_t - \delta k_t + sy_t$$
   where $0 < \delta < 1$ is the depreciation rate and $0 < s < 1$ is the saving rate. Assume that $sy_t = y_t^{1/2}$ in every period. Derive the first-order difference equation for the capital stock, sketch the phase diagram and determine whether the steady-state is stable or not.

3. Solve the following difference equations:
   a) $y_{t+2} - y_t = 0$
   b) $y_{t+2} + 2y_{+1} + y_t = 16$

4. Consider the following multiplier-accelerator model:
   $$Y_t = C_t + I_t + G$$
   $$C_t = mY_t$$
   $$I_t = a\ (Y_{t-1} - Y_{t-2})$$
   where Y is national income, C is consumption, I is investment, G is a constant level of government spending and 0 < m < 1 is the marginal propensity to consume. Derive and solve the second-order difference equation for national income. Determine what restrictions on the parameters of the model must be made to ensure convergence.