

EC351 Mathematical Economics
Autumn 2011 Test

Q1 Consider the following minimisation problem:

$$\underset{x,y}{\text{Min}} E = p_x x + p_y y \text{ subject to } x^\alpha y^{1-\alpha} = \bar{U}$$

where p_x, p_y are respectively, the prices, and x, y are the quantities of two goods, \bar{U} is the given level of utility that the consumer seeks to achieve while $0 < \alpha < 1$ is a parameter.

- (a) [10 marks] Write down the Lagrangean function for the minimisation problem and write down the first order conditions that characterise the solution;
- (b) [15 marks] Solve the first order conditions to find the optimal quantities, x^* and y^* , of the two goods. Provide an economic interpretation of your results.
- (c) [10 marks] Is the solution in (b) a local or a global minimum? Explain. Sketch the solution on a graph.
- (d) [10 marks] Use the Envelope Theorem to prove the Shephard's Lemma for this problem. Comment on your result.

Q2 Answer *all parts* of this question

(a) Consider the following maximisation problem:

$$\underset{x,y}{Max} (x + \alpha)(y + \beta)$$

subject to $px + qy \leq M$ and $x \geq 0, y \geq 0$

where p, q are the prices of x and y respectively, M is your income and α and β are positive constants that satisfy $\alpha p > \beta q$.

- (i) [10 marks] Write down the Kuhn-Tucker conditions for this problem. Are those conditions only necessary or are they also sufficient?
- (ii) [5 marks] Without doing any calculations how would you claim that the constraint should always bind at the solution of this problem?
- (iii) [15 marks] Find the solution of this problem when the constraint binds and $x^* > 0$ and $y^* > 0$. Under which assumption on the parameters do we have an interior solution for the problem? Explain.

(b) Consider the following market model:

The market demand is given by: $Q_t^D = \alpha - \beta P_{t-1}$ while the market supply is $Q_t^S = \delta + \varphi P_t$ where P is the price level, $\alpha > 0, \beta > 0, \delta < 0$ and $\varphi > 0$.

- (i) [5 marks] Find the first-order difference equation for the price level.
- (ii) [10 marks] Find the particular solution and the complementary solution of the equation in (i). Write down the general solution of that first order difference equation. Is the solution unique? Explain.
- (iii) [5 marks] Comment on the convergence of the general solution in (ii) to the steady-state.