ASSIGNMENT 4

Exercise 1 There are two firms, firm 1 and firm 2. Each firm has a constant marginal production cost which is equal to $c = 0$. Firms simultaneously choose prices.

There is a continuum of consumers. Each consumer wishes to buy one good and her maximum willingness to pay is $v > 0$. A fraction 1/2 of consumers observe the price charged by both firms. A fraction 1/4 of consumers observe only the price of firm 1 and the remaining fraction of consumers observe only the price of firm 2. A consumer can buy the product from firm i only if she observes the price of that firm.

1. Is there a pure strategy Nash equilibrium?

2. Derive the mixed strategy equilibrium of this game.

Solution Exercise 1

Suppose a Nash equilibrium in pure strategy exists, say $p_1 = p_2$. If $p_1 = p_2 = 0$, then firm 1 strictly gains by increasing her price. Suppose $p_1 = p_2 > c$. Then firm 1 strictly gains by slightly undercutting her price. Hence, if an equilibrium exists it is in mixed strategies.

Similar arguments as above, show that in a mixed strategy equilibrium firms price according to an atomless price distribution defined in a convex support with upper bound v. Simple calculation leads to a price distribution:

$$F(p) = 1 - \frac{1}{2}\frac{v-p}{p}$$

defined over the support $S = [1/3, v]$.

Exercise 2

There are two firms, firm 1 and firm 2, producing an homogenous good. The costs function is linear and the two firms have marginal costs $c = 0$. The inverse demand function is $p = a - Q$, where $Q = q_1 + q_2$, p denotes the price and q_i denotes the quantity produced by firm $i = 1, 2$, and a is a positive constant. Firms compete by setting quantities. The game is a two stages game. In the first stage, firm 1 chooses her quantity to produce. In the stage 2, firm 2 observes the quantity produced by firm 1 and then chooses her own quantity.

1. Derive the subgame perfect equilibrium of this game.

2. Compute the equilibrium profits of firm 1 and firm 2.

3 How does the difference in the equilibrium profits of firm 1 and firm 2 change with a?

Solution Exercise 2

1) By backward induction we can obtain that in the unique subgame perfect equilibrium we have that $q_1 = a/2$ and $q_2 = a/4$. Specifically, to solve for a subgame perfect equilibrium you need to follow the following steps. First, fix q_1, and find the optimal q_2. you then get a function $q_2(q_1)$ which tells you the optimal quantity that firm 2 will produce, given that firm 1 produces q_1. Then you move backward and you maximize profits of firm 1 given $q_2(q_1)$. By doing so you will get the desired outcome.

2) We can easily compute the equilibrium price, $p = a/4$. Then, the profits to firm 1 are $\pi_1 = \frac{a^2}{8}$, while the profits to firm 2 are $\pi_2 = \frac{a^2}{16}$.

3) Note that $\pi_1 - \pi_2 = \frac{a^2}{16} > 0$ and this difference is increasing in a. Note that firm 1 always obtains higher profits due to time asymmetries. Furthermore, the higher is the demand the more firm 1 gains relatively to firm 2.

Exercise 3

Consider the following coordination game.

$(Paul, John)$	L	R
L	$2, 2$	$0, 0$
R	$0, 0$	$3, 3$

1. Find the pure strategy Nash equilibria.

2. Define formally the notion of mixed strategy and then find symmetric mixed strategy equilibria of this game.

Solution Exercise 3

There are two pure strategy Nash equilibria. One is both players choose L, while the other is both players choose R.

A Mixed strategy of a player is a probability distribution over own action. In this game a mixed strategy of player i is simple a probability distribution over the set $\{L, R\}$. Let γ denote the probability that a player chooses L, while $1 - \gamma$ is the probability that a player would choose R. A mixed strategy Nash equilibrium is a γ^* such that each player is indifferent between choosing L with probability γ^* and R with the remaining probability.

Note that if Paul chooses L then he gets an expected utility of $2\gamma^*$, while if he chooses R he gets an expected utility of $3(1-\gamma^*)$. The indifference condition requires that: $2\gamma^* = 3(1 - \gamma^*)$, which implies that $\gamma^* = 3/5$.