Assignment 5: Advertising

Exercise 1

There are N firms selling a homogeneous good. The marginal cost is $c = 0$. There is one consumer that would like to buy the good. His reservation price is $v = 1$. The consumer does not know the existence of the firms. A firm can make the consumer aware of the product by advertising at a cost $\phi < 1$.

Firms simultaneously choose whether to advertise the consumer and the price to charge.

Derive the Symmetric Nash Equilibrium of this game.

Solution First note that a necessary condition for equilibrium is that firms randomize in advertising. Suppose not. Then there are two possibilities. One, firms stay out with probability 1. Then I want to go in and charge $p = 1$. Suppose firms are all in with probability 1. Then $p = 0$, but then firms face a negative profit.

Consider firm i, and assume that all other firm advertise with probability λ. The profit of firm i when she advertises a price p is

$$E\pi_i(p;\lambda) = p\sum_{k=0}^{n-1}\binom{n-1}{k}\lambda^k(1-\lambda)^{n-1-k}[1-F(p)]^k - \psi$$

and this expression can be simplified as follows

$$E\pi_i(p;\lambda) = p[1-\lambda F(p)]^{n-1} - \psi$$

Note that the upper bound of the price distribution must be $p = 1$. Thus, $E\pi_i(1;\lambda) = 1 - \lambda - \psi$. In equilibrium a firm must be indifferent between advertising and staying out of the market. That is: $E\pi_i(1;\lambda) = 0$ if and only if $1 - \lambda - \psi = 0$ if and only if $\lambda = 1 - \psi$.

Finally, in equilibrium, a firm that advertise must be indifferent between advertising any price in the support of the price distribution. That is: $E\pi_i(p;\lambda) = 0$ for all $p \in \sigma$, where σ is the support. Solving you obtain

$$F(p) = \frac{1}{\lambda} - \frac{1}{\lambda}\left(\frac{\psi}{p}\right)^{n-1}$$

The lower bound of the equilibrium distribution, say p_l is then obtained by solving $E\pi_i(p_l;\lambda) = 0$.