

Assignment 4: Entry

Exercise 1 In this game, depending on his type, player 2 either wants to coordinate with 1, or he wants to not coordinate. Specifically, consider the following Bayesian game. There are two players: $N = \{1, 2\}$. The actions of player 1 are $A_1 = \{up, down\}$, the actions of player 2 are $A_2 = \{L, R\}$. Player 1 is of type $T_1 = \{+\}$, while player 2 can be of two types $T_2 = \{+, -\}$. It is common knowledge that $P(+, +) = \pi$ and $P(+, -) = 1 - \pi$, where $\pi \geq \frac{2}{3}$. The payoffs are given by the following two payoff matrices:

$(t_1 = t_2 = +)$	L	R
up	1, 1	0, 0
down	0, 0	2, 2

$(t_1 = -t_2)$	L	R
up	0, 1	2, 0
down	1, 0	0, 2

1. Write a pure strategy of player 1 and player 2 and define the expected payoffs of each player for that particular strategy.
2. Define the Bayesian pure strategy equilibria of this game.

solution

$(Down; Right|+, Left|-)$

$(Up; Left|+; Right|-)$

Exercise 2:

There are two firms: firm 1 (the incumbent) and firm 2 (the potential entrant). Production costs are linear, and Firm 1 marginal cost is $\bar{c}_1 = 2$ with probability q and $\underline{c}_1 = 1/2$ with probability $1 - q$. Firm 2 has a marginal costs of $c_2 = 3/2$. At each period the inverse market demand is $p = 10 - Q$, where p is the market price when total production is Q .

Firms are playing the following game. In the first stage nature choose the marginal cost of firm 1. This is observed by both firms. In the second stage, firm 1 is alone in the market and chooses a quantity to produce. In the third stage firm 2 observes the quantity produced in the previous stage and decide whether to enter the market or not. In the fourth stage, both firms know whether firm 2 has entered the market or not. If firm 2 has entered the market, firms compete a la Cournot. Otherwise, firm 1 is a monopolist.

- 1 represent the game using a tree. Define a strategy of firm 1 and a strategy of firm 2.
- 2 Define the notion of a subgame perfect equilibrium
- 3 Find a subgame perfect equilibrium of this game.

Exercise 3

There are two firms: firm 1 (the incumbent) and firm 2 (the potential entrant). Production costs are linear, and Firm 1 marginal cost is $\bar{c}_1 = 2$ with probability q and $\underline{c}_1 = 1/2$ with probability $1 - q$. Similarly, Firm 2 marginal cost is $\bar{c}_2 = 2$ with probability v and $\underline{c}_2 = 3/2$ with probability $1 - v$. These probability distributions are common knowledge. At each period the inverse market demand is $p = 10 - Q$, where p is the market price when total production is Q . Consider the following game.

In the first stage Nature chooses the marginal costs of firm 1. Firm 1 observes his marginal cost. In the second stage, firm 1 chooses a quantity. In the third stage, firm 2 observes the quantity of firm 1 and her marginal costs (but not the marginal costs of firm 1) and decides whether to enter the market or not to enter the market. If firm 2 enters the market, she pays $k = 7$. In the fourth stage, the marginal costs of both firms become common knowledge. If firm 2 has entered the market, the two firms compete a la Cournot. If firm 2 did not enter the market, firm 1 chooses a quantity as a monopolist.

- I Suppose that in the third stage firm 2 observes the marginal costs of the firm 1 before deciding if entering the market or not. Show that in this situation, regardless of firm 2's marginal costs, firm 2 will never enter the market.
- II Find a Pooling equilibrium.
- III Find a separating equilibrium.
- IV compare the two equilibria.

It is better that I tell them to show that something is a pooling equilibrium.