Solution Exam Ec366– 2010-2011

Question 1.

PART a.

(a_1) The student should define the utility of a buyer located at x who buys from A, i.e., $v - p_A - t(x-a)^2$, and the utility of a buyer located at x who buys from B, i.e., $v - p_B - t(x-(1-b))^2$. Then, the student should solve for the indifferent consumer, leading to:

$$\tilde{x} = \frac{a+(1-b)}{2} + \frac{p_B - p_A}{2t[a-(1-b)]}.$$

Finally, the student should argue that the demand of A is given by all consumers located at the left of the indifferent consumer, i.e., $D_A(p_A, p_B) = \tilde{x}$. The Demand of B is $D_B(p_A, p_B) = 1 - \tilde{x}$.

(a_2) The student should define the utility of i, $i = A, B$, for arbitrary p_A and p_B, i.e., $\Pi_i(p_A, p_B) = D_i(p_A, p_B)p_i$. Derive the best replies of each pub: the best reply of A is $t + p_B - 2p_A = 0$ and the best reply of B is $t + p_A - 2p_B = 0$. Solving, $p_A = p_B = t$.

(a_3) The student should show that the demand of i is like in a standard Bertrand: $1/2$ if prices are the same, 1 if the price of i is the lowest, and 0 in the remaining case. Given this, the student should simply show that the unique equilibrium is $p = 0$.

PART b. The student should state that the incentives to initiate a price war, that is undercutting the rival's price, decreases if the new potential demand cannot be serve, and this is the case when the firm is operating near to her capacity constraints. A good student will provide the answer using some formal analysis.

Question 2.

(a_1) The student should show that if $p_1 > p_2$ then the demand of producer 1 is $1/3$, if $p_1 < p_2$ then the demand of producer 1 is $2/3$ and if $p_1 = p_2$ then the demand of producer 1 is $1/2$.

(a_2) We have developed the proof more times during the course in similar setting. An excellent answer should elaborate on the following steps. Fist, show that in equilibrium firms cannot charge a price with positive probability. By contradiction, if they do, they will tight with at that price with strictly positive probability and therefore reducing slightly the price increases profits. Second, the support of prices in the mixed equilibrium must be convex. If it is not convex, then there exists a price p, which is lower than the upper bound, it is in the support and such that a slightly higher price is not anymore in the support. When a firm charges this price, its profit

can increase by charging a slightly higher price. Indeed, by doing so, the demand of the firm does not change, but the per-consumer profit is higher.

Given these two observations, the student should define the expected utility of a firm charging a price, given the other firm is pricing according to a distribution F defined in a convex support σ, i.e.,

$$E\Pi(p) = p\frac{1}{3} + p\mu[1 - F(p)]$$

The student should now note that the upper bound of the support must be v. So, $E\Pi(v) = v\frac{1}{3}$. Finally, the student should impose the indifferent conditions for mixed Nash equilibrium and obtain the desire price distribution, i.e., $E\Pi(p) = E\Pi(v)$, i.e.,

$$F(p) = 1 - \frac{v - p}{p}$$

Finally, the student should derive the lower bound of the support, i.e., $\frac{v}{2}$

(b) A good answer should be refer to models of consumer search and price dispersion covered in the lecture in order to provide theoretical argument supporting the claim. Furthermore, the student should refer to the article of Sorensen (2000) published in JPE as empirical support of the claim.

Theoretical support: the information that consumers have is essential in shaping the level of competition between firms. When consumers are not aware of all prices, we have seen that firms, when pricing, trade-off high per-consumer profit and low demand with low per=consumer profit and high demand, and this generates price dispersion: firms randomize in price and so, theoretically, one can observe different prices for similar product. Of course, the level of dispersion will depend on how much consumers are informed, which depend on how much they search. In market of repeated purchase the benefit of searching for a consumer is higher because the purchase must be incurred for many periods, and therefore in these markets we should expect that consumers are on average more informed. Theoretically this means that competition is stronger and therefore there is less price dispersion.

Empirical: the paper of Sorensen (2000) provides support for the claim. He is able to test the hypothesis above because he compares level of price dispersion across different drugs. Certain drugs must be taken for a short period, while other for repeated period. Using this distinction and collecting data about prices of different drugs in different pharmacy, Sorensen shows that in general there is price dispersion and that the level of price dispersion is lower for drugs which require repeated purchase.

Question 3.

(a_1) This is a Prisoner's Dilemma game and the unique Nash equilibrium is (L, L)

(a_2) If the game is repeated for a finite time, by backward induction, it is easy to show that the unique equilibrium strategy is: play L in every period regardless of the history. A student should elaborate on the argument using backward induction.

(a_3) A possible trigger strategy is: Start play H and keep playing H as long as one player deviates; otherwise play L. Assuming that players sum and discount stage payoffs, say with discount δ, this equilibrium strategy is equilibrium whenever $200/(1-\delta) \geq 250 + 50\delta/(1-\delta)$, which is satisfied whenever δ is sufficiently high.

(a_4) During the lectures we have discuss different aspects that make collusion more difficult. For example: uncertainty about the state of the demand, imperfect information about prices charged, different discount factors of firms. A good student should mention different factors that makes collusion more difficult and should elaborate at least for one aspect in more details.

Question 4.

(a)

(a_1) There are two equilibria: (I, NI) and (NI, I).

(a_2) The equilibrium is the following: firm A plays I, firm B plays NI if I and I if NI.

(a_3) the game in part i, due to the simultaneous move, generates multiplicity of equilibria. The theory predicts that only one firm introduce the new product, but does not predict who will do that. On the other hand, in part b, the fact that firm A chooses first, allows her to have an advantage. A good student would briefly mention the first mover advantage idea that we discussed in the course.

(b) The student should refer to the article and model of Iyer et al.(2005) published in Marketing Science. The model of that paper has been developed during the course and it has been commented. The article Iyer et al. (2005) is in the reference list of the course and students are aware that it is important material of the course.

Briefly, the model of Iyer et al.(2005) is as follows: there are two firms, a unit mass of consumers with inelastic demand and reservation price r. A mass h of consumer is loyal to firm 1, an equal mass to firm 2, and a mass $s = 1 - 2h$ is price sensitive. Consumers can only buy if they receive some ads. Two specifications are studied:

1. Firms cannot target advertising. Here, firms choose simultaneously whether to advertise or not and the price. If a firm advertises the cost is A and all consumers observe the ads. It is shown that in equilibrium firms randomize in advertising and pricing. Firms obtain zero profit.

2. Firms can target advertising. The game is the same as above, but now a firm can: advertise to the loyal group, to the price sensitive group or to both. Firms

cannot price discriminate. In equilibrium it is shown that: firms advertise to their loyal segment and randomize in the other segment. Pricing is in mixed strategy.

Using the two characterization, it is possible to compare what is the role of targeting advertising on total advertising. In the paper the following conclusions are obtained, and this is the core of the answer to the exam question:

A. When advertising is relatively inexpensive, in the non-targeted model firms advertise a lot and this creates a lot of waste: each firm waste money in reaching consumer who are loyal to the other firm. In this case, targeting advertising allow firms to reduce this waste. Therefore, in this case, technology which allows firm to target advertising should lead to lower level of advertising.

B. When advertising is relatively expensive, in the non-targeted model firms advertise little. Introducing targeted technology in this case increases total advertising. By targeting advertising the marginal value of ads increases and therefore firms will advertise more, even if it is very expensive.

Question 5.

(a)CHECK CALCULATION

(i) The student should solve the game by backward. So, first start from firm B; its profit is

$$\Pi_B = q_B(10 - q_A - q_B - c_B)$$

taking the first order condition and solving for q_B we have:

$$q_B = 5 - \frac{q_A}{2} - \frac{c_B}{2}$$

plugging this strategy in firm A's profit we have

$$\Pi_A = q_A[5 - \frac{q_A}{2} + \frac{c_B}{2}]$$

taking first order condition we have

$$q_A = 5 + \frac{c_B}{2}$$

So, the subgame perfect equilibrium is given by the strategy profile: $q_A = 5 + \frac{c_B}{2}$ and $q_B = 5 - \frac{q_A}{2} - \frac{c_B}{2}$. The equilibrium outcome is: $q_A = 5 + \frac{c_B}{2}$ and $q_B = 5 + \frac{3c_B}{4}$.

(ii) Given the equilibrium outcome, the price in equilibrium is

$$p = \frac{5}{2} + \frac{c_B}{4}$$

and it is easy to solve

$$\Pi_A = \left(\frac{5}{2} + \frac{c_B}{4}\right)\left(5 + \frac{c_B}{2}\right)$$

and

$$\Pi_B = \left(\frac{5}{2} - \frac{3c_B}{4}\right)^2$$

(iii) the student should verify that the profit of firm A is higher than the profit of firm B, and the difference in profit is increasing in c_B. The more inefficieny is firm B relative to firm A, the higher is the profit that firm A is able to extract.

(b). Limit pricing refers to a situation in which an incumbent firm sets low prices (or produce high quantity) in order to deterred entry. During the lecture we have discussed why limit pricing cannot be conceived as an equilibrium outcome in the presence of complete information. Formally, while limit pricing is a Nash equilibrium, it is not a subgame perfect equilibrium. That is, limit pricing is based on a non credible threats. Further in the lecture we have discussed that in a word of incomplete information limit pricing can be obtained as an equilibrium outcome. The students should illustrate the model: an incumbent can be high or low cost, the incumbent knows her type, by the potential entrant does not. The potential entrants has some prior, which are updated by observing the price charged by the incumbent in the first stage. In this model, a separating equilibrium can be obtained when the difference between high cost and low cost is sufficiently high. In this case, an efficient incumbent firm can credible signal to the potential entrant that she is efficient (by setting a sufficiently low price) and this will deter entry.