Solution Mid Term Exam-EC366 2010-2011

Solution Exercise 1.
Question 1

Firm 1 and firm 2 choose whether to enter either market I or market U. If both firms enter market I they obtain a payoff of 3 each. If both firms enter market U they obtain a payoff of 5 each. If firms enter a separate market they obtain a payoff of π each, where π is a number greater than 0.

1 Suppose firms play a one-shot game in which they choose the market to enter.

1.1 [10 points] Represent the game as a normal-form game;

1.2 [10 points] Define the set of action profiles of this game;

1.2 [20 points] Find the pure strategy Nash equilibrium of this game, depending on the value of π.

2 Now, suppose that firms play a two-stage game. In the first stage firm 1 chooses to enter either market I or market U. In the second stage, firm 2 observes the decision taken by firm 1, and then firm 2 decides whether to enter either market I or market U. Payoffs are the same as in part 1 above.

2.1 [10 points] Represent the game as a tree.

2.2 [10 points] Define the set of action profiles and write down all the pure strategies of firm 1 and all the pure strategies of firm 2.

2.3 [20 points] Find the pure strategy subgame perfect Nash equilibrium, depending on the value of π.

3 [20 points] Compare and discuss the results obtained in point [1.3] and point [2.3].

1.1. The game in normal-form is

$(Firm1, Firm2)$	I	U
I	$3,3$	π,π
U	π,π	$5,5$

1.2. The action set of firm i is $A_i = \{I, U\}$, $i = 1, 2$. The set of action profiles is $A = A_1 \times A_2 = \{II, IU, UI, UU\}$.

1.3. If $\pi < 3$ there are two Nash equilibria: (I, I) and (U, U). If $3 < \pi < 5$ there is one Nash equilibrium (U, U). If $\pi > 5$ there are two Nash equilibria (I, U) and (U, I).

Solution Part 2.

2.1. It is easy to represent the game as a tree.

2.2. The action profiles as the same as in point 1. The strategies of firm 1 are $S_1 = \{I, U\}$. Each strategy of firm 2 prescribes firm 2 what to play for every possible action of firm 1. Thus $S_2 = \{(x|I, y|U), x, y \in \{I, U\}\}$. For example a strategy $s_2 = (I|I, U|U)$ means that firm 2 enter market I when firm 1 has entered market I, while firm 2 enters market U if firm 1 has entered market U.

2.3. If $\pi < 3$ then the subgame perfect equilibrium is given by the following strategy profile. Firm 1 plays U and firm 2 plays strategy $(I|I, U|U)$. The outcome is both firms enter market U. If $3 < \pi < 5$ then the subgame perfect equilibrium is given by the following strategy profile. Firm 1 plays U and firm 2 plays strategy $(U|I, U|U)$. The outcome is again both firms enter market U. Finally, if $\pi > 5$ there are two subgame perfect equilibria. One subgame perfect equilibrium is given by the strategy: firm 1 plays U and firm 2 plays $(U|I, I|U)$. The outcome in this case is firm 1 enters market U and firm 2 enter market I. The other subgame perfect equilibrium is given by the strategy: firm 1 plays I and firm 2 use the following strategy $(U|I, I|U)$. In this case the outcome is firm 1 enters market I and firm 2 enters market U.

3. Observe that subgame perfect equilibria are a subset of Nash equilibria. This is because of the definition of subgame perfect equilibrium, which requires that a strategy profile is a Nash equilibrium in the all game and that it is also a Nash equilibrium in every subgame. If $\pi < 3$, there are two Nash equilibria, while there is only one Subgame perfect equilibrium. The reason for this is that the Nash equilibrium (I, I) is based on a non-credible threat and therefore if we have a sequential game, it does not survive the refinement of subgame perfection.