Solution Exercises: Short run competition

Solution Exercise 1

The action profiles of this game are

$$\{(Bach, Bach), (Stravinsky, Bach), (Bach, Stravinsky), (Stravinsky, Stravinsky)\}$$

There are two pure strategy Nash equilibria. These are: $(Bach, Bach)$ and $(Stravinsky, Stravinsky)$.

Solution Exercise 2

The normal-form game representation is

$(1,2)$	Head	Tail
Head	$1,0$	$0,1$
Tail	$0,1$	$1,0$

The action profiles are: $\{(Head, Head), (Head, Tail), (Tail, Head), (Tail, Tail)\}$

There is not a pure strategy Nash equilibrium.

Solution Exercise 3

a) Firm 1's inverse demand is

$$p = a - b\sum_{j=2}^{N} q_j - bq_1$$

The profit to firm 1 is

$$\pi_1 (q_1, ..., q_N) = q_1 \left(a - b\sum_{j=2}^{N} q_j - bq_1 \right)$$

First order condition:

$$\frac{\partial \pi_1}{\partial q_1} = a - b\sum_{j=2}^{N} q_j - 2bq_1 = 0$$

Thus, the reaction function is

$$q_1^* (q_2, .., q_n) = \frac{a}{2b} - \frac{1}{2}\sum_{j=2}^{N} q_j$$

(b) To solve for a symmetric Nash equilibrium we apply symmetry. Let the Cournot equilibrium be $q_1 = q_2 = ... = q_N = q^C$. Then q^C solve

$$q^C = \frac{a}{2b} - \frac{1}{2}(N-1)\,q^C$$

which yields to

$$\begin{aligned} q^C &= \frac{a}{b(N+1)} \\ Q &= \frac{Na}{b(N+1)} \\ p &= \frac{a}{(N+1)} \end{aligned}$$

(c) As the number of firms increases, output and price approach the competitive limit.