EC366
Assignment 2: Long-Run Competition

Question 1. Consider the following Prisoner's Dilemma game

Paul\John	C(cooperate)	D(defect)
C(cooperate)	5,5	0,10
D(defect)	10,0	1,1

(a) Suppose Paul and John play this game only once. Which is the Nash equilibrium;

(b) Suppose Paul and John play this game repeatedly for 2 times.

- (b_1) Represent the game in extensive form (as a tree). Describes all the information sets of Paul and John. Describes all the subgames. Write a possible strategy of Paul.
- (b_2) Solve for the subgame Nash equilibrium? (Use Backward Induction)

(c) Suppose Paul and John play this game for an infinite period of time. Let $\delta \in [0,1]$ be the discount factor of Paul and John.

- (c_1) Show that playing always defection can be sustained as a subgame perfect equilibrium.
- (c_2) Define a trigger strategy profile in order to support cooperation in equilibrium
- (c_3) Using the trigger strategy you have defined in (c_2) find the range of δ for which cooperation can be sustained in a subgame perfect equilibrium.

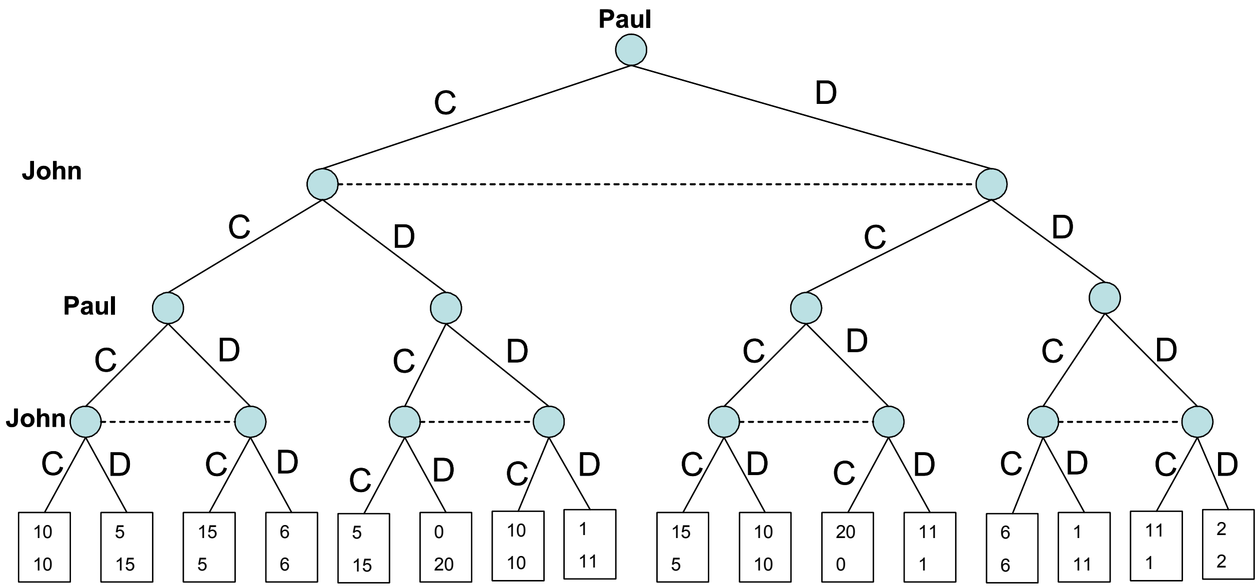
(d) Interpret all the results you have found in term of the collusive behaviour between firms competing in prices.

Solution
(a) In the one-shot Prisoner's dilemma game the Nash equilibrium is (D, D).

We first show that (D, D) is a Nash equilibrium:
Suppose Paul believes that John plays D, then if Paul plays C he obtains 0, while if he plays D he obtains 1. Thus, the best response of Paul given his believes on the behaviour of John is to play D. The same argument can be done for John. This shows that D, D is a Nash equilibrium.

We now must show that (D, D) is the unique Nash equilibrium:
Consider the candidate (C, D). This is not a Nash equilibrium because given that Paul believes that John will play D, he will strictly prefer to play D as well.
Consider the candidate (C, C). In this case it is easy to see that Paul will prefer to deviate and to play D, given that John plays C.

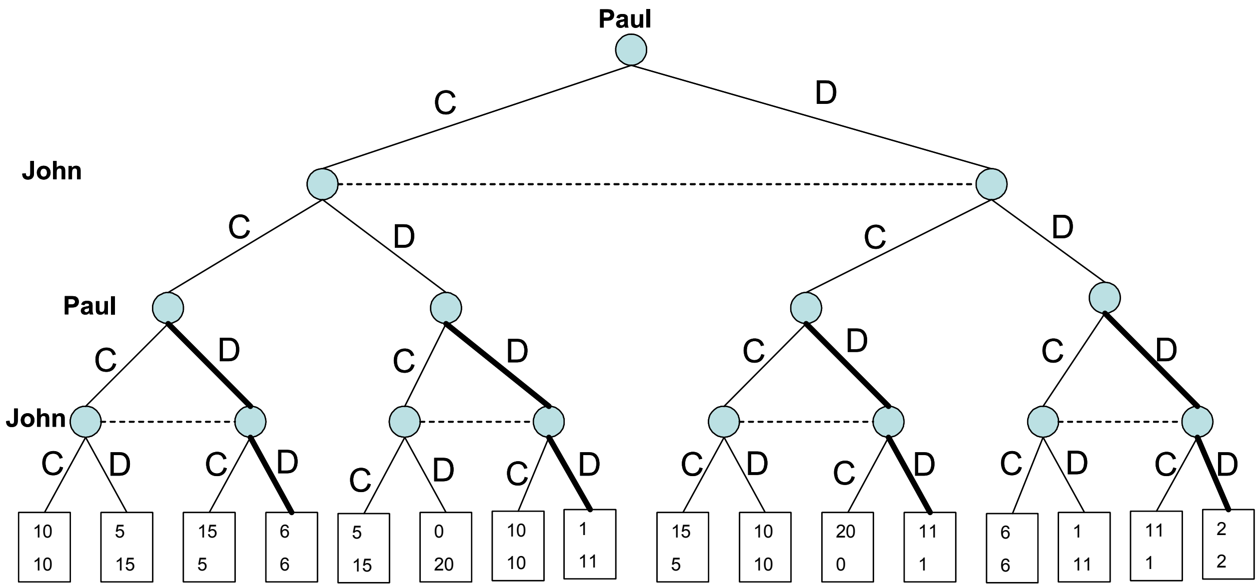
(b) This is the extensive representation of this game



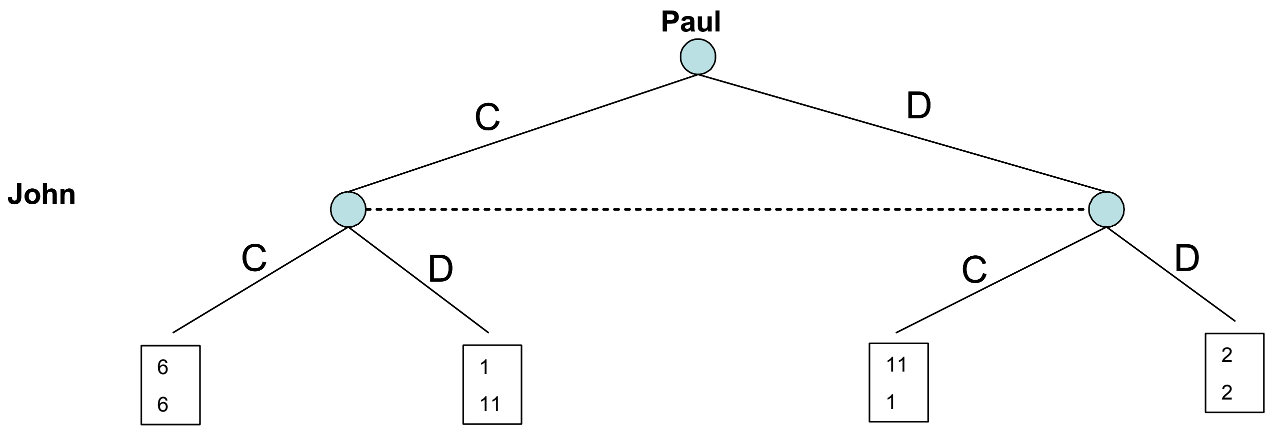
See lecture note 4 to identify the subgames and the information sets.

A possible strategy for Paul is:
To play C in the first period
And in the second period:
to play D if in the first period the realization was (D, D) or (C, C)
to play C if in the first period the realization was (D, C) or (C, D)
Note that this is just one possible strategy for Paul. You see that the strategy is specified for any possible history of the game.

The unique subgame Nash equilibrium outcome is mutual defection in the first period and mutual defection in the second period.
To show this we must solve the game backwards. We start from the end of the game and we solve the Nash equilibrium in each subgame.



We see that in each subgame Paul and John are playing a one-shot Prisoner's dilemma game. And we know that the unique Nash equilibrium is mutual defection. We then move upward. Since we know the Nash equilibrium in each subgame following the first period we can simplify the tree as follows:



Or we can represent the game in the usual form:

Paul\John	C(cooperate)	D(defect)
C(cooperate)	6,6	1,11
D(defect)	11,1	2,2

It is easy to see that the unique Nash equilibrium here is (D, D).

Thus, the unique subgame perfect equilibrium outcome is D, D in the first period and D, D in the second period.

(c) We now consider that the game is played for an infinite number of periods.

(c_1) To show that to play always defection is an outcome which can be sustained as a subgame perfect equilibrium is trivial. Indeed, consider a strategy profile which prescribes to both John and Paul to play defection for any possible history. Since to play (D, D) is a Nash equilibrium of the constituent game it will be a Nash equilibrium in any possible subgame. And therefore it can be sustained in a subgame perfect equilibrium.

(c_2) We define the following standard trigger strategy

Strategy for Paul:

Paul starts to play C in the first period, $t = 1$;
Paul plays C at period τ if he observes that till that point both Paul and John have played C;
Otherwise Paul plays D.

Strategy for John:

John starts to play C in the first period, $t = 1$;
John plays C at period τ if he observes that till that point both Paul and John have played C;
Otherwise Paul plays D.

(c_3) To find the range of δ in order that cooperation can be sustained as a subgame perfect equilibrium we need to proceed as follow.

First: we assume that John follows the strategy we have constructed in (c_2).

Second: we define the payoff of Paul in two possible scenario.

Scenario one: Paul follows the strategy profile too. In this case we will have that both Paul and John will play cooperation forever. This means that the utility to Paul will be

$$\begin{aligned} u_{\text{Paul}} &= 5 + 5\delta + 5\delta^2 + ... \\ &= \sum_{t=0}^{\infty} 5\delta^t = 5\frac{1}{1-\delta} \end{aligned}$$

Paul gets 5 in each period but you must take into account the discount factor δ.

Scenario two: Paul in the first period deviates. In this case Paul will get 10 in the first period, but from period 2 onwards he will get 1 because both Paul and John will defect forever. Therefore the utility to Paul in case he deviates is:

$$\begin{aligned} u^d_{\text{Paul}} &= 10 + 1\delta + 1\delta^2 + ... \\ &= 10 + \sum_{t=1}^{\infty} \delta^t \\ &= 10 + \frac{1\delta}{1-\delta} \end{aligned}$$

Third: the last step is to compare the utilities to Paul in the two scenario. For an equilibrium we want that Paul does not have an incentive to deviate. This means that we need to solve:

$$u_{\text{Paul}} \geq u_{\text{Paul}}^{d}$$

Using the expression of the utilities, we have that:

$$\begin{aligned} 5\frac{1}{1-\delta} &\geq 10+\frac{1\delta}{1-\delta} \\ 5 &\geq 10\left(1-\delta\right)+\delta \\ \delta &\geq \frac{5}{9} \end{aligned}$$

Thus, when δ is sufficiently high, i.e. $\delta \geq \frac{5}{9}$, mutual cooperation can be sustained in equilibrium.