EC372 Economics of Bond and Derivatives Markets

Mock Exam 2012

Answer Guidelines

Section A

1. A bond with face value \$100 will mature one year from the present, at which time the holder will receive the face value plus a single coupon of \$32. Define and calculate the bond's spot yield if its market price today is \$120.

   Answer guide:

   The bond's spot yield, y, is defined to satisfy:

   $$p = \frac{m + c}{(1 + y)}$$

   where p is the bond's price, m is its face value and c is the coupon. Hence, with $p = 120$, $m = 100$, $c = 32$:

   $$p = 120 = \frac{132}{(1 + y)}, \quad \text{so that } y = \frac{132}{120} - 1 = 0.10$$

   Hence, the spot yield on the bond is 10%.

2. Define and interpret the 'Macaulay duration' for a coupon-paying bond with n years to maturity.

   Answer guide:

   The Macaulay duration, D, for a bond with n years to maturity is defined by

   $$D = \frac{1}{p}\left(\frac{1 \cdot c}{(1 + y)} + \frac{2 \cdot c}{(1 + y)^2} + \cdots + \frac{n \cdot (c + m)}{(1 + y)^n}\right)$$

   where p is the bond's market price, c is its coupon, m is its face value, n is the number of years to maturity, and y is the yield to maturity.

   The Macaulay duration is a measure of the responsiveness of the bond's price with respect to changes in its yield to maturity. The units of measurement are 'years', such that $D < n$ for $c > 0$.

3. Suppose that the spot price of wheat today is \$8.00 per bushel and that the risk-free interest factor for the next 6 months is 1.10. Making any additional assumptions you need, show how to obtain the forward price of wheat for delivery 6 months from today in the absence of arbitrage opportunities.

Answer guide:

Assume that: markets are frictionless, storage costs are zero, the convenience yield of holding wheat is zero and ample stocks of wheat exist. Let F denote the forward price of wheat for delivery 6 months from today.

Suppose $F > 1.10 \times 8 = 8.80$. Then arbitrage profits can be made by borrowing cash to buy spot wheat, which is stored and sold forward. After 6 months the wheat is delivered in return for the forward price, the value of which exceeds the amount that is repaid on the loan.

Suppose that $F < 1.10 \times 8 = 8.80$. Then arbitrage profits can be made by borrowing wheat, selling it spot, investing the funds (at the risk-free rate), and buying the wheat forward. After 6 months, the payoff on the investment exceeds the amount needed to pay for the wheat, which is received and returned from whomever it was borrowed.

Hence, the forward price in the absence of arbitrage opportunities equals: $F = 1.10 \times 8 = 8.80$.

4. Explain what is meant by 'marking-to-market' in the context of futures contracts.

   Answer guide:

   Marking-to-market refers to the process by which assets are revalued at the current market price, the gains (losses) being credited (debited) to the investor's margin account held with the investor's broker.

   In the context of futures contract, suppose that one contract is purchased at t with delivery date T for price $f(t, T) = 50$. If $f(t+1, T) > 50$, the margin account is credited with $f(t+1, T) - 50$. If $f(t+1, T) < 50$, the margin account is debited with $50 - f(t+1, T)$. It is as if each day, the contract is sold and repurchased at the current price.

   Similarly, if the initial transaction is to *sell* the contract, marking-to-market is as if the contract is purchased and re-sold each day, the gain (loss) being credited (debited) to the margin account.

5. Describe a 'long perfect hedge' strategy, using an example with a forward (or futures) contract as the hedge instrument.

   Answer guide:

   A 'long hedge' refers to the reduction of price risk with respect to a commitment to *purchase* an asset (commodity) at a specified date in the future. A 'perfect' hedge is one for which the price risk is reduced to zero.

   To achieve a 'long perfect hedge', a forward contract is made to *purchase* the asset for delivery at the specified future date for a known forward price today. At the delivery date, the asset is received in return for the agreed forward price.

Exactly the same result is achieved if (and only if) the asset underlying the futures contract is to be purchased on the delivery date specified in the futures contract.

6. What are the differences between 'European' and 'American' style 'call' and 'put' option contracts?

   Answer guide:

   A 'European' option can be exercised only at its expiry date, while an 'American' option can be exercised at any time prior to or at the expiry date.

   A European *call* option is an option to purchase an asset at its expiry date for the exercise (strike) price stipulated in the contract. If the option is American the asset can be purchased at any time no later than the expiry date for the exercise price.

   A European *put* option is an option to *sell* an asset at its expiry date for the exercise (strike) price stipulated in the contract. If the option is American the asset can be *sold* at any time no later than the expiry date for the exercise price.

7. Explain how the payoff at expiry on a European put option with exercise price 200p varies with the underlying asset price, for an investor who *writes* the option when the premium equals 15p.

   Answer guide:

   If the underlying asset price at expiry is greater than or equal to 200p, the option is allowed to die unexercised, with a net payoff (gain) to the writer of 15p (the option's premium).

   If the underlying asset price at expiry, S_T, is less than 200p, the option is exercised, the asset being sold to the writer for 200p. The gross payoff from the option is thus a *loss* of $200 - S_T$, i.e., the exercise price at which the writer is obliged to purchase the asset minus its market price. The net payoff is $15 - (200 - S_T)$, which is positive if $S_T > 185$.

   See figure 1 on page 4.

8. Describe, using a diagram, the relationship between the premium on a *European call* option and the price of the underlying asset.

   Answer guide:

   See Figure 2 on page 4. In the Absence of Arbitrage Opportunities (AoAO), the option premium for each level of S must be no higher than the upper dashed line, i.e., $c < S$, and no lower than the lower dashed, line, i.e., $c \geqq \max[0,\ S - X/R]$.

   With the benefit of additional assumptions (about the distribution of S), it is, in principle, possible to predict the option price as shown by the solid line, which is drawn for a given exercise price, X, interest factor R, time to expiry and asset price volatility.

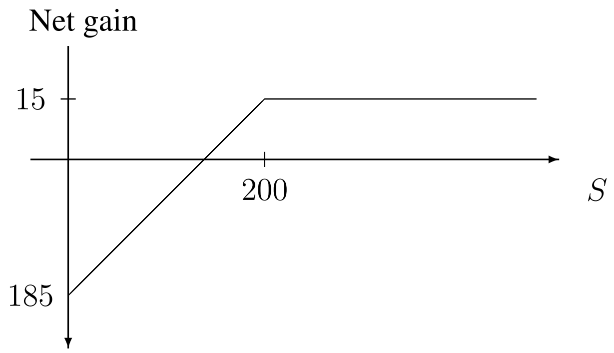

Figure 1: Payoff at exercise for a put option: short position

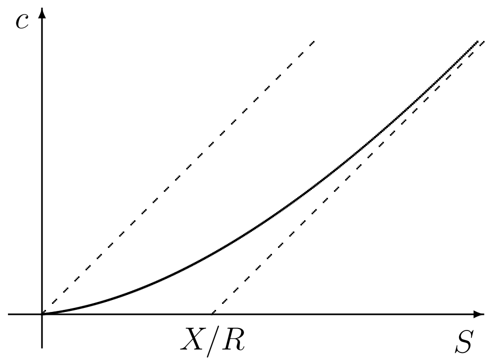

Figure 2: Call option price as a function of the asset price S.

9. What is meant by 'portfolio insurance'? Give an example of how it can be achieved.

   Answer guide:

   'Portfolio insurance' refers to a class of investment strategies that seek to place a floor under the value of a portfolio at some specified future date, T. Let S_T denote the market value of the portfolio at date T and S_I its 'insured' value. The payoff on the insured portfolio at T is given by $S_P \geq S_I$. See figure 3 on page 5.

   The portfolio insurance shown in figure 3 can be achieved by the purchase of a put option on the portfolio expiring at T with exercise price X. The premium on the put option is given by $X - S_I$.

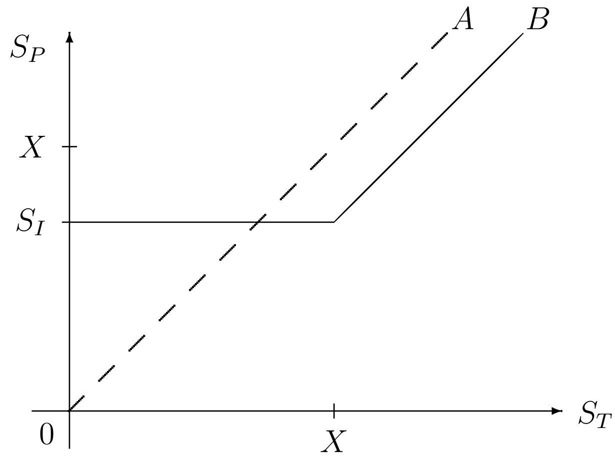

Figure 3: Portfolio Insurance with a put option

10. Describe, using an example, a *plain vanilla* interest rate swap.

    Answer guide:

    A 'plain vanilla' interest rate swap involves the exchange of a sequence of fixed interest payments in return for a floating interest rate payments for a specified period of time.

    Example: A fixed rate of 5% per annum of a notional principal equal to £10m is exchanged at six-monthly intervals for a period of 7 years in return for LIBOR+20b.p. on the same notional principal at the same dates. Only the net interest flow changes hands at each date. The notional principal is not exchanged.

11. Explain how a *swap agreement* can be interpreted as a 'bundle of forward contracts'.

    Answer guide:

    - Discarding the inessential details, a swap is an agreement between two parties, A and B, to undertake a sequence of transactions at specified dates, say every 6 months, for a determinate period of time, say 5 years.
    - Each separate transaction (there are 10 in the above example) can be interpreted as a forward contract, i.e. a contract made at the outset of the swap for execution at the specified date. Hence a swap can be interpreted as a 'bundle of forward contracts'.
    - Answers may provide an example, though this is not obligatory if the explanation is clear. A typical example would be a plain vanilla interest rate swap of a floating rate for a fixed rate. Suppose that A pays a fixed rate to B in return for a floating rate. Then it is as if A has taken the 'long' side of a sequence of forward agreements, while B has taken the 'short' side.

12. Briefly explain why ‘moral hazard’ may arise in the relationship between a financial intermediary and the firms (borrowers) to which it lends.

    Answer guide:

    ‘Moral hazard’, by definition, arises when the principal in a relationship has incomplete control over an agent (on whom the principal relies to fulfil the terms of a contract).

    In this context, the ‘principal’ – a financial intermediary, say a ‘bank’ – lends to an agent – a borrower, say a firm – over which the bank may have incomplete control, i.e. the firm’s managers may be able to take actions (such as enjoying ‘private benefits’ or otherwise shirking in their efforts) that are either hidden from the bank or, even if not hidden, over which the bank has limited power to stop.

    The existence of the private benefits or the capacity to shirk, suggests that the bank and the firm have different objectives: the bank (principal) may wish to maximise its payoff from the loan, while the firm may wish to sacrifice (in favour of private benefits or the opportunity to shirk) part of the expected payoff from the investment funded, partly at least, by the bank.

    Thus the two crucial ingredients of why ‘moral hazard’ may arise are: (a) that the bank has incomplete control over its borrowers (formally that there are ‘incomplete contracts’); and (b) that the bank and its borrowers have different objectives.

Section B

13. Two zero-coupon bonds, labelled N (nominal) and R (real) each mature four years from today. At maturity, bond N pays £100. At maturity, bond R pays £100 adjusted for the increase in the price index of goods and services between today and the maturity date, i.e. the payoff will be increased to take account of inflation.

    (a) [20 marks] Define and interpret the following four yields:

    (i) nominal and real spot-yields on bond N;

    (ii) nominal and real spot-yields on bond R;

    Answer guide:

    Notation (for generality use n for the time to maturity):

	Spot yields	
	Nominal	Real
Nominal ZC bond	y_n	y_n^*
Real ZC bond	$\widetilde{y}_n$	$\widetilde{y}_n^*$

Nominal ZC bonds:

The nominal spot yield on a nominal ZC bond is its *yield to maturity*, the rate of return that is obtained if the bond is held to maturity. Hence, y_n satisfies:

$$p_n = \frac{m}{(1+y_n)^n} \quad \text{that is:} \quad p_4 = \frac{100}{(1+y_4)^4}$$

where p_n is today's market price of N and $m = \$100$ is its face value.

The *real* spot yield on a nominal ZC bond is its yield to maturity, where the payoff, \$100, is adjusted ('deflated') for changes in the price level of goods and services between today and the maturity date. Suppose that price price level increases by a factor Z_4.

Hence, the real payoff on the nominal ZC bond is $m/Z_n = 100/Z_4$ — the nominal \$100 has been 'deflated' by the rise in the price level. Thus, y_n^* is defined to satisfy:

$$p_n = \frac{m/Z_n}{(1+y_n^*)^n} \quad \text{that is:} \quad p_4 = \frac{100/Z_4}{(1+y_4^*)^4}$$

Real ZC bonds:

The *real* spot yield on a real ZC bond is its yield to maturity, the rate of return that is obtained if the bond is held to maturity, omitting the requirement to increase the face value according to changes in the price level (i.e., as if the price level remains constant). Hence, $\widetilde{y}_n^*$ satisfies:

$$\widetilde{p}_n = \frac{m}{(1+\widetilde{y}_n^*)^n} \quad \text{that is:} \quad \widetilde{p}_4 = \frac{100}{(1+\widetilde{y}_4^*)^4}$$

The *nominal* spot yield on a real ZC bond is its yield to maturity, where the payoff, \$100, is adjusted ('inflated') for changes in the price level of goods and services between today and the maturity date.

Hence, the nominal payoff on the rate ZC bond is $m \times Z_n = 100 \times Z_4$ — the nominal \$100 has been 'inflated' by the rise in the price level. Thus, $\widetilde{y}_n$ is defined to satisfy:

$$\widetilde{p}_n = \frac{m \times Z_n}{(1+\widetilde{y}_n)^n} \quad \text{that is:} \quad \widetilde{p}_4 = \frac{100 \times Z_4}{(1+\widetilde{y}_4)^4}$$

(b) [15 marks] Being careful to state your assumptions, discuss the inferences that could be drawn from the spot-yields in (a) for the average rate of inflation over the coming four years.

Answer guide:

The assumption that enables inferences to be drawn about expected inflation is that the nominal, or real, yield on N equals that on R. This condition might itself be deduced from an assumption that investors are risk-neutral or hold point ('certain') expectations about future inflation. Given that the bonds are identical except with regard to expected inflation, they should yield the same return after adjusting for inflation expectations.

The prediction is that $\widetilde{y}_4 = y_4$, or equivalently that $\widetilde{y}_4^* = y_4^*$. Thus:

$$(1+\widetilde{y}_4)^4 = (1+y_4)^4 \tag{1}$$

$$\frac{100 \cdot Z_4}{\widetilde{p}_4} = \frac{100}{p_4} \tag{2}$$

$$\frac{\widetilde{p}_4}{p_4} = Z_4 \tag{3}$$

Hence, under the stated assumption, the price level is predicted to increase by a factor equal to the ratio of the bond prices, minus 1: $(\widetilde{p}_4/p_4) - 1$.

(c) [5 marks] Suppose that today's market price of bond N is £75 while today's price of R is £90. In view of your answer to (b), what, if anything, does this information imply about the change in the price index of goods and services over the coming four years?

Answer guide:

Substituting the stated values:

$$\frac{\widetilde{p}_4}{p_4} = Z_4 \tag{4}$$

$$\frac{90}{75} = Z_4 \tag{5}$$

$$Z_4 = 1.20 \tag{6}$$

Thus the price level is expected to increase by a factor of 20% over the next 4 years. This at an annual rate of approximately 4.66%.

(d) [10 marks] Suppose that an investor expects the price index of goods and services to increase from 360 today to 450 four years from today.

Discuss how, if at all, the investor could profit from this expectation by trading in N and R. Does the investor face an 'arbitrage opportunity'? Explain.

Answer guide:

The investor expects the price level to increase by a factor of
$25\% = (450/360) - 1$.

If expectations are held with certainty, in frictionless markets the following strategy would result in an expected profit (for zero initial outlay):

1. Issue (sell) one N bond for 75 today.
2. Use the proceeds to buy 75/90 R bonds.
3. After 4 years the investor expects the payoff on the R bonds to equal: $\frac{75}{90} \times 100 \times 1.25 \approx 104.167$
4. After 4 years, the investor redeems the N bond at a cost of 100
5. Net gain ≈ 4.167.

This is not an arbitrage opportunity unless there exists a 'forward market' for trading in the price level, i.e. the value of Z_4 is an expectation, not known until after four years.

14. Answer *both* parts (a) and (b).

(a) [28 marks] Define, and discuss the implications of, the following motives for trading in futures markets:

(i) Arbitrage,
(ii) Speculation,
(iii) Hedging.

Answer guidelines:

- Arbitrage: arbitrageurs seek to exploit price differentials among spot and futures prices in order to make arbitrage profits (risk-free payoffs that require a zero initial outlay). Market equilibrium is usually defined such that arbitrage opportunities are absent, i.e. that arbitrageurs have successfully exploited any such opportunities, with the unintended consequence that their collective actions have driven arbitrage profits to zero.
- Speculation: speculators seek to profit by trading according to their expectations about the future. They bear the risk that their expectations may turn out to be wrong.
- Hedging: hedgers trade in futures markets to reduce the risks associated with other production or merchandising commitments. For example, a grain merchant may wish to sell wheat forward – adopt a '*short* position' – in order to guard against the possibility that the value of the stored grain will have fallen by the date at which it is sold. A miller, on the other hand, may

wish to buy forward – adopt a '*long* position' – in order to guard against a rise in the price of grain before it is needed in the milling process.

- Answers (of a 2.1 or 1st class standard) should also comment on the implications of the various motives:
  - Arbitrage serves to link the present spot and futures prices. In the simplest case (zero storage costs and convenience yields): $f(t, T) = R(t, T)p(t)$, where $p(t)$ is the spot price at t, $f(t, T)$ is the futures price at t for delivery at T, and $R(t, T)$ is the interest factor for borrowing/lending between t and T.
  - The influence of hedging depends on the relative importance of long and short hedging: long (short) hedging tends to drive the futures price up (down) relative to expectations of the spot price at the delivery date.
  - Speculators tend to bear the risks that the hedgers seek to to avoid. But this need not mean that speculation passively responds to hedging pressures. If there are strongly held expectations about the level of future spot prices, these are likely to have a dominant impact on current futures contract prices (so that hedgers are more 'passive').

(b) A manufacturer, for which copper is an input, seeks to hedge against fluctuations in the spot price of copper that it needs to acquire several months from the present. Assume that a market exists for copper futures contracts.

(i) [12 marks] Explain how the manufacturer could use futures contracts to accomplish the hedge.
Answer guidelines:

- The manufacturer is a 'long hedger', i.e. would take a long position in the futures markets, buying copper contracts for delivery near to the date at which the physical copper is to be acquired.
- As spot and futures prices for copper are likely to become ever closer as the futures delivery date approaches, if copper prices increase in the intervening months, the manufacturer will be able to sell the futures contracts at a higher price than at which they were purchased. Thus there will be a positive payoff on the futures contracts, which offsets (at least partially) the increase in the spot price of copper.
- If copper prices fall in the months prior to the firm taking delivery of the copper, although a loss will be made on the futures contracts, the firm will buy copper at a low spot price.
- Thus, the uncertainty about the future spot price of copper is reduced through the purchase of futures contracts. The hedge does not, however, imply that the manufacturer will get the best of both worlds: protection against increasing prices but only pay low prices if copper prices fall. This is not the function of hedging (which is to control the cost, not to place a floor under it).
- It is possible for the manufacturer to eliminate the price risk altogether by holding the futures contracts to delivery. This will be appropriate if

the firm wishes to acquire the grade of copper specified in the contract, on the date, and at the place, also specified in the contract.

- Some answers may wish to express the argument more formally, as follows. Let p_1 denote the spot price of copper at the date, $t = 1$, in the future when the copper is to be acquired, while f_0 denotes the futures contract price today, $t = 0$, and f_1 is the futures contract price at date $t = 1$ (assuming that the futures contracts mature later than date 1). Also let N denote the amount of copper to be acquired, and M the number of futures contracts purchased.

  Then the net cost of copper, C, with hedging is:

  $$C = p_1 N - (f_1 - f_0)M$$

  In the extreme case of a perfect ('risk-free') hedge, the firm takes delivery using the futures contract, choosing $M = N$, in which case $C = f_0 N$: all price risk has been eliminated.

(ii) [10 marks] Why is the manufacturer's hedging strategy unlikely to be entirely risk-free? How could the risks be minimised?
Answer guidelines:

- The main reason why the strategy is unlikely to be risk-free is that the manufacturer may not wish to take delivery of the copper according to the terms of the futures contract:
  - The copper may be of a different grade (standard of refinement, etc) than stated in the contract.
  - The copper may be needed before or after the delivery date.
  - The copper may be needed in a different location than than stated in the contract.
  - Tailing the hedge: during the life of the futures contracts, the firm will be required to deposit funds in a margin account. While the funds normally earn interest at a market rate, uncertainty about fluctuations in interest rates will create additional uncertainty about the ultimate payoff on the hedging strategy.
- The analysis supposes that the manufacturer has determined at the outset the amount of copper to be acquired. If it turns out that more, or less, copper than originally planned is to be acquired, then the futures position cannot exactly offset fluctuations in copper prices.
- The risks associated with hedging can be minimised if the firm chooses a hedge instrument (futures contract), the price of which tends to be most highly correlated with the spot price of copper that the firm will acquire. Although the correlation will not generally be perfect, the relationship between the two should enable estimation of a 'hedge ratio', which links the number of futures contracts purchased to the amount of copper to be acquired – in the presence of risk the optimal hedge ratio may be greater or less than unity. [Answers may provide detail

about the construction of a risk-minimising hedge ratio but this is not obligatory, even for first class answers.]

15. Answer both (a) and (b).

(a) [25 marks] The shares of Blanko plc currently trade for 205 pence each. European style call and put options on Blanko's shares are also traded, both with one period to expiry and both with an exercise price of 210 pence per share. The current option prices, per share, are observed to be 25 pence for each call and 10 pence for each put. Assume: the risk-free interest rate is 5% per period; interest is compounded only once per period; Blanko plc pays no dividends over the coming period; markets are frictionless.

Use an arbitrage argument to show that a risk-free payoff using zero initial capital can be obtained at the security prices given above. Hence, state the put-call parity relationship for European style options. [A formal derivation of the put-call parity relationship is not required.]

Answer guide:

Answers may choose to state the put-call parity relationship at the outset:

$$c + \frac{X}{R} = S + p,$$

where c is the European call premium, p is the European put premium, X is the option exercise price, R is the interest rate factor, and S is the underlying asset price.

The information given does not satisfy the put-call parity relationship:

$$c + \frac{X}{R} = 25 + \frac{210}{1.05} = 225 > 215 = 205 + 10 = S + p.$$

Hence, consider the following strategy: write one call, buy one put, buy one unit of stock and borrow 190:

	Initial cash flow	Outcome: $S_T > 210$	Outcome: $S_T \leq 210$
Write one call option:	$+25$	$-(S_T - 210)$	0
Buy one put option:	-10	0	$210 - S_T$
Buy stock:	-205	$+S_T$	$+S_T$
Borrow:	$+190$	-199.50	-199.50
Net total	0	$+10.50$	$+10.50$

In the event that the share price at expiry, S_T exceeds the option exercise price, the call is exercised with a loss of $S_T - 210$ the put option dies unexercised, the stock is worth S_T, and the loan is repaid with interest for 199.50, leaving a net gain of $210 - 199.50 = 10.50$.

In the event that the share price at expiry, S_T is less than the option exercise price, the call dies unexercised, the put option is exercised with a gain of $210 - S_T$, the stock is worth S_T, and the loan is repaid with interest for 199.50, leaving a net gain of $210 - 199.50 = 10.50$.

Hence, a risk-free gain of 10.50 has been made (i.e. irrespective of the share price at the options' expiry date) with a zero initial-outlay. By definition, this is an arbitrage profit.

(b) Suppose that the underlying security for call and put options is a futures contract. Assume that the call and put options are both of European style, with the same exercise price and delivery date.

(i) [20 marks] Describe the distinctive characteristics of options contracts in which the underlying security is a futures contract. Illustrate your answer with examples, one for a call option, the other for a put option.
Answer guide:

- Distinctive characteristics of options on futures:
  1. The option is to take a long (call) or short (put) position in a futures contract at the option exercise price.
     The relationship between options and the assigned futures position can be summarised as follows:

	Call options	Put options
Long futures	long call (holder)	short put (writer)
Short futures	short call (writer)	long put (holder)

     The *writer* of options on futures receives the option premium in return for an obligation to take a futures position opposite to the buyer in the event that the option is exercised.

  2. Options normally expire at, or very shortly before, the futures contract delivery date (would not make sense for options to expire *after* the futures contract delivery date).

- Example 1: American call option on oil futures
  - Buy one option with $X = \$30$ on futures with December delivery
  - Each futures contract is for 1000 barrels of oil
  - Suppose that: $f(\textit{September}, \text{December}) = \$34$
  - Suppose option is *exercised* in September:
    1. Option payoff (gross of premium): $\$4000 = 1000 \times (34 - 30)$
    2. One futures contract is *received* from option writer
  - In frictionless market: futures contract could be sold without loss
  - In a frictionless market: payoff from selling option $\geq$ exercise

- Example 2: American put option on gold futures
  - Buy one option with $X = \$400$ on futures with December delivery

  - Each futures contract is for 100 troy ounces of refined gold
  - Suppose that: $f(\textit{October}, \text{December}) = \$390$
  - Suppose option is *exercised* in October:
    1. Option payoff (gross of premium): $\$1000 = 100 \times (400 - 390)$
    2. Option writer sells one futures contract on behalf of option holder
  - In frictionless market: could buy futures contract without loss
  - In a frictionless market: payoff from selling option $\geq$ exercise

- With market frictions (transaction costs), it may be profitable to exercise the option, and to retain the futures contract (to offset later or deliver)

(ii) [5 marks] How, if at all, should the put-call parity relationship be modified when the security underlying the options is a futures contract? [A formal derivation is not required.]
Answer guide:

- Put-call parity for European options:

$$c + \frac{X}{R} = p + S$$

- For European options on *futures*:

$$c + \frac{X}{R} = p + \frac{f}{R}$$

- Notice presence of f/R, not f
- The reason is that f is paid only at T (delivery date) while payment is typically made for other other assets at the time of purchase or sale.

End of Paper

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