

EC372 Economics of Bond and Derivatives Markets

Option Pricing

How to Derive an Option Price Formula

1. Make assumptions, most importantly about: (i) frictionless markets, (ii) about determination of the underlying asset price in the future.¹
2. Propose an arbitrage portfolio, comprising options, the underlying asset and risk-free borrowing (or lending): (a) zero initial outlay, and (b) non-negative payoff in every eventuality (state of the world).
3. In market equilibrium, the arbitrage portfolio has a **zero** payoff in every state.²
4. Derive an option price formula from the conditions that define an arbitrage portfolio together with the absence of arbitrage opportunities:

$$c = f(S, X, \tau, R, \sigma)$$

(see chapter 19 of *Economics of Financial Markets* for notation).

Numerical Example

1. Assume: frictionless markets, that today's underlying asset price, $S = 80$; that S will change after one time period to exactly one of two values $uS = 128$ (state 1) or $dS = 48$ (state 2); that the interest factor for borrowing or lending is $R = 1.20$; that there is a call option on the underlying asset with exercise price $X = 88$, expiring after one time period.
2. An arbitrage portfolio satisfies: buy M options, buy N units of the underlying asset, borrow B , such that:

$$\text{Zero outlay: } -cM - 80N + B = 0. \quad (1)$$

$$\text{Payoff in state 1: } (128 - 88)M + 128N - 1.20B \geq 0.$$

$$\text{Payoff in state 2: } (0)M + 48N - 1.20B \geq 0.$$

3. In market equilibrium (absence of arbitrage opportunities):

$$\text{Payoff in state 1: } (128 - 88)M + 128N - 1.20B = 0. \quad (2)$$

$$\text{Payoff in state 2: } (0)M + 48N - 1.20B = 0. \quad (3)$$

4. Solve (1), (2) and (3) for $c = 20$.

¹Assumptions are abstractions and, of necessity, unrealistic. Why make them? Because without assumptions, there would be no implications (i.e. no option price formula). The art – the successful application of which wins Nobel prizes – is to make assumptions that are tolerable approximations to the world *and* which permit insightful implications to be deduced from them.

²There are at least two other ways of expressing the absence of arbitrage opportunities — they are all equivalent. See page 469 of *Economics of Financial Markets*.