EC372 Economics of Bond and Derivatives Markets

American and European Put Option Premiums

This note seeks to clarify why the price (premium) of an American put option almost always exceeds that of a European put option, even in frictionless markets.[1]

Let P_t and p_t denote the respective prices of American and European put options on the same asset, with exercise price X, expiring at date T, where $t < T$. Remember that $P_t \geqq p_t$ for all $t < T$: the opportunity to exercise the American option before T could never negative value (because the only difference between the American and European styles is that the American option could be exercised early, an opportunity that is not available for the European option).

Suppose, then, that $P_t = p_t$, at some date $t < T$.[2] Consider the strategy: "write one European put and buy one American put". Given that $P_t = p_t$, the outlay is zero. The payoff on this strategy is never negative and could be positive: hence it is an arbitrage opportunity. More precisely, in no state of the world is the payoff negative, and there may exist states for which he payoff is positive.

To understand *why* the strategy provides an arbitrage opportunity, note that exactly one of the following outcomes must occur:

1. $P_t = p_t$ for *all* $t < T$ (the American option never becomes worth more than the European). Then at expiry, $t = T$, either both options die or both are exercised. In each case, the payoff is zero: nothing is gained but nothing is lost.
2. $P_s > p_s$ for some $s < T$. Then offset both options (sell an American option and buy a European option), for an immediate gain of $P_s - p_s > 0$.

   But is it conceivable that $P_s > p_s$?[3] Yes, if the underlying asset price, S, falls low enough. Remember that, in the absence of arbitrage opportunities, $P_t \geqq X - S_t$ and $p_t \leqq X/R$, for all $t \leqq T$. Suppose that, for some date s, $S_s < \widehat{S}$, where $\widehat{S}$ is such that $X - \widehat{S} = X/R$. Then $P_s > p_s$.

Hence, if $P_t = p_t$ nothing is to be lost from the strategy "write one European put and buy one American put", and in some states the payoff may be positive. Hence, it cannot be excluded that $P_t > p_t$ even when $S_t \geqq \widehat{S}$ (because there may exist some state for which the underlying asset price falls so low that $S_t < \widehat{S}$ before the option expires).

Are there *any* circumstances in which $P_t = p_t$, at *every* date $t \leqq T$? Yes, but only if all investors agree that the underlying asset price will never fall below $\widehat{S}$, defined above (formally: all investors believe that there is *no* state for which $S_t < \widehat{S}$ at any date $t \leqq T$).

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[1] In the presence of market frictions, anything could happen depending on the nature of the frictions. However, if any frictions impinge on American and European contracts in the same way and to the same extent, it is plausible to argue that the opportunity for early exercise of an American option would never impose an *extra* cost compared with a European option. This being so, the reasoning in the note is not undermined by the presence of market frictions.

[2] Note that, at expiry, $t = T$, the payoff on the American option is identical with that on a European option, namely, $\max[0,\ X - S_T]$, so that $P_T = p_T = \max[0,\ X - S_T]$.

[3] Formally: can there exist at least one state for which $P_s > p_s$?