Term Structure of Interest Rates

Yield curves

Yield curves show the relationship between **spot yield** and **time to maturity** (i.e. 'life') of a bond.

Spot yield: $y_{n,t} \equiv (m/p_{n,t})^{(1/n)} - 1$ is the yield to maturity on a Zero Coupon (pure discount) bond with life n at date t, where $p_{n,t}$ is the market price of the bond at date t and m is its face value. *From now on all bonds are assumed to be Zero Coupon.*

Theories of the term structure seek to explain the shape of the yield curve at each date.

Implicit forward rates

Implicit forward rates are future yields (interest rates) implied by today's observed bond yields.

Define ${}_{n-1}f_n$ as the implied rate between dates $n-1$ and n as of today, t:

$$(1 + y_{n-1})^{n-1}(1 + {}_{n-1}f_n) = (1 + y_n)^n \tag{1}$$

All the above are defined as of today, i.e. should have an extra t subscript.

The term structure (yield curves) can equivalently be expressed using implicit forward rates, rather than spot yields.

Expectations hypothesis of the term structure

The expectations hypothesis claims that expectations about future bond yields determine the shape of the yield curve.

Heroic assumption: bond prices on all future dates are treated as known, or are expected with certainty, as of today, date t.

Example:

$$(1 + y_{2,t})^2 = (1 + y_{1,t})(1 + y_{1,t+1}) \tag{2}$$

Four equivalent forms of the (pure) expectations hypothesis (*Economics of Financial Markets*, pp. 319–21):

1. *Local Expectations Hypothesis*: the expected one year rates of return on all bonds are equal to the spot yield observed on 1-year bonds.

$$y_{1,t} = \frac{p_{n-1,t+1}}{p_{n,t}} - 1 = \frac{(1 + y_{n,t})^n}{(1 + y_{n-1,t+1})^{n-1}} - 1 \tag{3}$$

   The LEH asserts equality of the 1-year holding-period yields on all bonds.

2. *Return to Maturity* (RM-EH): the expected return to \$1 invested for n years is the same irrespective of bonds in which the funds are invested. See the example above, (2), or for $n > 2$, as

$$(1 + y_{n,t})^n = (1 + y_{1,t})(1 + y_{1,t+1})(1 + y_{1,t+2}) \cdots (1 + y_{1,t+n-1}) \tag{4}$$

3. *Yield to Maturity*: the yield to maturity on \$1 invested for n years is the same irrespective of the bonds in which the funds are invested. In the example, (2) is replaced with

$$y_{2,t} = \{(1 + y_{1,t})(1 + y_{1,t+1})\}^{1/2} - 1 \tag{5}$$

For n-year bonds:

$$y_{n,t} = \{(1 + y_{1,t})(1 + y_{1,t+1})(1 + y_{1,t+2}) \cdots (1 + y_{1,t+n-1})\}^{1/n} - 1 \tag{6}$$

4. *Unbiased Expectations* (UB-EH): implicit forward rates equal expected yields. In the above example: ${}_1f_2 = y_{1,t+1}$, where ${}_1f_2$ is the one-period forward rate in the second period from the present.

Implication (prediction): all bonds have the same holding period yield for any specified holding period, e.g., the LEH becomes:

$$\ldots = r_{n+1,t+1} = r_{n,t+1} = r_{n-1,t+1} = \ldots = r_{2,t+1} = r_{1,t+1} \tag{7}$$

where $r_{n,t+1}$ is the holding period yield on an n-year bond between dates t and $t + 1$, and $r_{1,t+1} \equiv y_{1,t}$ is the spot yield on 1-year bonds, i.e. 'the rate of interest'.

Problem: future bond yields are unknown (i.e. uncertain). Hence, replace future bond yields with their *expectations*. But why?

Note: the four forms of the expectations hypothesis are no longer equivalent when future bond yields are uncertain.

Liquidity preference (liquidity premium, risk premium) theory

Allows for 'liquidity premiums' (or risk, or term, premiums):

$$\mathrm{E}_t[r_{n,t+1}] - \ell_n = \mathrm{E}_t[r_{n-1,t+1}] - \ell_{n-1} = \ldots = \mathrm{E}_t[r_{2,t+1}] - \ell_2 = r_{1,t+1} \tag{8}$$

where the term premium on an n-year bond is defined as $\ell_n = \mathrm{E}_t[r_{n,t+1}] - r_{1,t+1}$.

Hicks's theory: $\ell_n > \ell_{n-1} > \ldots > \ell_2 > 0$.

Preferred habitat (hedging pressure) theory

This theory refines the liquidity preference theory to allow for differing preferences among lenders and borrowers with respect to the maturity of the bonds they hold or issue.

Main prediction: expectations of future interest rates are not exclusively responsible for the pattern of current bond yields, i.e., stocks of bonds, and investors' demands to hold them, also influence the term structure.

Segmented markets hypothesis

The segmented markets hypothesis is an extreme variant of the preferred habitat theory. It asserts that the markets for bonds of different maturity are *segmented* (i.e. separate). Implication: demands and supplies for bonds of different maturities determine their prices (yields) – no reason for relationships among yields on bonds with different maturities.

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