University of Essex
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EC501 Econometric Methods and Applications

Problem Set 1

The Classical Linear Regression Model

1. Consider the following regression models:

$$\begin{array}{ll} \text{Model 1:} & y_i = \beta_1 + \epsilon_i, \quad i = 1, \ldots, n. \\ \text{Model 2:} & y_i = \beta_2 x_{i2} + \epsilon_i, \quad i = 1, \ldots, n. \\ \text{Model 3:} & y_i = \beta_1 + \beta_2 x_{i2} + \epsilon_i, \quad i = 1, \ldots, n. \end{array}$$

   In the above, x_{i1} and x_{i2} are (possibly non-random) regressors, and the random disturbance ϵ_i satisfies $E(\epsilon_i|x_{i1}, x_{i2}) = 0$, $E(\epsilon_i^2|x_{i1}, x_{i2}) = \sigma^2$, and $E(\epsilon_i\epsilon_j|x_{i1}, x_{i2}) = 0$ for $i \neq j$. For each of these models:

   (a) derive an expression for the least squares estimator by minimizing the sum of squares function;

   (b) check that the second-order conditions for a minimum are satisfied;

   (c) derive the expected value of the least squares estimator.

2. Consider the following regression models:

$$\begin{array}{ll} \text{Model 1:} & y_i = \beta_1 + \epsilon_i, \quad i = 1, \ldots, n. \\ \text{Model 2:} & y_i = \beta_1 + \beta_2 x_{i2} + \epsilon_i, \quad i = 1, \ldots, n. \\ \text{Model 3:} & y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i, \quad i = 1, \ldots, n. \\ \text{Model 4:} & y_i = \beta_1 + \beta_2 i + \epsilon_i, \quad i = 1, \ldots, n. \end{array}$$

   In the above, x_{i1} and x_{i2} are (possibly non-random) regressors, and the random disturbance ϵ_i satisfies $E(\epsilon_i|x_{i1}, x_{i2}) = 0$, $E(\epsilon_i^2|x_{i1}, x_{i2}) = \sigma^2$, and $E(\epsilon_i\epsilon_j|x_{i1}, x_{i2}) = 0$ for $i \neq j$. Each of the above models is a special case of the model

$$\mathbf{y} = \mathbf{X}\beta + \epsilon$$

   where $\mathbf{y}$ is $n \times 1$, $\mathbf{X}$ is $n \times K$, β is $K \times 1$ and ϵ is $n \times 1$.

   (a) Write down the matrix $\mathbf{X}$ and find $\mathbf{X}'\mathbf{X}$ and $\mathbf{X}'y$ in each case in terms of the sums of squares and cross-products of the variables.

   (b) Using the results in part (a), obtain expressions for the OLS estimators of the following parameters:
   (i) β_1 in Model 1; (ii) β_2 in Model 2; (iii) β_1 in Model 3; (iv) β_2 in Model 4.

   (c) Describe briefly the differences in the type of behaviour that Models 1 and 4 imply for y_i.

   (d) What interpretation may be given to the residuals in Models 1 and 4?