University of Essex — Department of Economics
Session: 2011/12 — João Santos Silva

EC501 Econometric Methods and Applications

Problem Set 3

Inference in the Classical Linear Regression Model

1. In the model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \epsilon_i, \quad i = 1, \ldots, n,$$

a sample of 100 observations yields the following sample moments (where $\sum$ denotes $\sum_{i=1}^{n}$):

$$\sum x_{i2}^2 = 150 \quad \sum x_{i2} = 120 \quad \sum x_{i2} y_i = 9$$
$$\sum y_i^2 = 74 \quad \sum y_i = 10$$

(a) Compute the ordinary least squares estimates of β_1 and β_2.

(b) Compute the covariance matrix of the vector, b, of ordinary least squares estimates of $\beta = (\beta_1 \ \beta_2)'$.

(c) Conduct a test of the hypothesis that $\beta_2 = 0$ against the alternative that $\beta_2 \neq 0$ at the 5% level of significance.

(d) What does the answer to part (c) suggest about the relationship between y and x_2?

2. In the model

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \epsilon_i, \quad i = 1, \ldots, n,$$

it is assumed that $\epsilon_i \sim NID(0, \ \sigma^2)$. The following sample moment matrix is obtained from a sample of 63 observations.

	y	x_1	x_2	x_3
y	10	2	1	0
x_1		4	−1	0
x_2			4	1
x_3				2

For example, $\sum x_{i1} y_i = 2$ and $\sum x_{i1} x_{i2} = -1$. Furthermore, we know that

$$(X'X)^{-1} = \frac{1}{26} \begin{bmatrix} 7 & 2 & -1 \\ 2 & 8 & -4 \\ -1 & -4 & 15 \end{bmatrix}$$

(a) Obtain estimates of β_1, β_2, and β_3, and calculate their standard errors.

(b) Using a 5% level of significance, test the hypothesis that $\beta_1 = 0$ against that alternative that $\beta_1 \neq 0$.

(c) Using a 5% level of significance, test the hypothesis that $\beta_1 = 0$ and $\beta_3 = 0$ against the alternative that at least one of β_1 and β_3 is non-zero.

3. (You may wish to revisit this question after the computing classes have started). The Stata file `Problem_Set_03_Data.dta` contains data relating to output prices (y), labour costs (x_2) and import costs (x_3) for the U.K. over the period 1959–1994 (all variables are in index form normalised with a value of 1 in 1990). Estimation by OLS of the model

$$\log y_i = \beta_1 + \beta_2 \log x_{i2} + \beta_3 \log x_{i3} + \epsilon_i,$$

using the sample period 1960–1994, lead to the following results:

```
                                     Number of obs =        35
                                     F(  2,     32) =30523.24
-----------------------------------------------------------
          ly |      Coef.   Std. Err.       t    P>|t|
-------------+---------------------------------------------
         lx2 |   .9816755   .0464554    21.13   0.000
         lx3 |   .0313708   .0402388     0.78   0.441
       _cons |   .0020919   .0058119     0.36   0.721
-----------------------------------------------------------
```

Knowing that

$$s^2(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 0.00215810 & -0.00186195 & 0.00007662 \\ -0.00186195 & 0.00161916 & -0.00005078 \\ 0.00007662 & -0.00005078 & 0.00003378 \end{bmatrix},$$

test of the following hypotheses:

(a) $\beta_2 = 1$;

(b) $\beta_3 = 0$;

(c) $\beta_2 + \beta_3 = 1$;

(d) $\beta_2 = 0$ and $\beta_3 = 0$.

(e) What (economic) interpretation can be given to the results in parts (a)–(c)?

4. Consider the two regression models

$$\begin{aligned} y &= X\beta + \epsilon & (1) \\ y &= X\beta + Z\gamma + \epsilon & (2) \end{aligned}$$

where y and ϵ are $n \times 1$, X is $n \times K_1$, Z is $n \times K_2$, β is $K_1 \times 1$ and γ is $K_2 \times 1$. Further, assume that $E(\epsilon|X, Z) = 0$ and $E(\epsilon\epsilon'|X, Z) = \sigma^2 I_n$.

(a) Show that, in the model (2),

$$\hat{\beta} = (X'M_Z X)^{-1} X' M_Z y$$

where $M_Z = I - Z(Z'Z)^{-1}Z'$.
Hint: Consider the estimated model $y = X\hat{\beta} + Z\hat{\gamma} + e$. To derive $\hat{\beta}$, pre-multiply this equation by $X'M_Z$ and use results on the symmetric idempotent matrix M_Z to simplify.

(b) Under the assumption that (2) is correctly specified, what are the properties of $\hat{\beta}$ and $\hat{\gamma}$?

(c) Under the assumption that $E(y|X,Z) = X\beta$, what are the properties of $\hat{\beta}$ if (2) is estimated?

(d) Under the assumption that $E(y|X,Z) = X\beta + Z\gamma$, what are the properties of the OLS estimator b of β if (1) is estimated?