University of Essex
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EC501 Econometric Methods and Applications

Problem Set 4

Large Sample Methods

1. Find the limits (if they exist) as $n \to \infty$, of the following deterministic sequences:

   (a) $x_n = n^{-2}$;
   (b) $x_n = (-1)^n$;
   (c) $x_n = 4 - 2/n$;
   (d) $x_n = e^{-n}$;
   (e) $x_n = \ln 2^n$;
   (f) $x_n = n^{-2}(n+2)(n-3)$.

2. A random sample of independent observations $y_1, \ldots, y_n$ is drawn from a distribution whose probability density function is given by

$$f(y) = \beta \exp\{-\beta y\}, \quad y > 0,$$

   where β is an unknown positive parameter.

   (a) Obtain an expression for the maximum likelihood estimator, $\hat{\beta}$, of β.
   (b) Given that in a sample of size 50 we observe $\sum_{i=1}^{50} y_i = 25$, calculate the maximum likelihood estimate of β.
   (c) Derive the asymptotic distribution of $\hat{\beta}$.
   (d) Using the data given in part (b), conduct a test of the hypothesis that $\beta = 1$ against the alternative that $\beta \neq 1$ at the 5% level, using a t-test based on the asymptotic distribution.

3. The continuous random variable x has probability density function given by

$$f(x;\theta) = \left(\frac{1}{2\pi\theta}\right)^{1/2} \exp\left\{-\frac{x^2}{2\theta}\right\},$$

   where $-\infty < x < \infty$ and $\theta > 0$.

   (a) Find the maximum likelihood estimator $\hat{\theta}$ of θ based on a random sample of independent observations $x_1, \ldots, x_n$.
   (b) Derive the asymptotic distribution of $\hat{\theta}$. (Note that: $x \sim N(0, \theta)$.)
   (c) A sample of 100 observations yielded $\sum_{i=1}^{n} x_i^2 = 110$. Compute $\hat{\theta}$.
   (d) Conduct a test of the hypothesis that $\theta = 1$ against the alternative that $\theta \neq 1$ at the 5% level using a likelihood ratio test.