

EC501 Econometric Methods and Applications

Problem Set 5

Large Sample Methods (continued)

1. In the linear regression model $y = X\beta + \epsilon$, large sample theory usually requires an assumption that

$$\text{plim} \frac{X'X}{n} = Q,$$

where Q is a finite nonsingular nonrandom matrix. Is this assumption satisfied in the linear trend model $Y_i = \beta_1 + \beta_2 i + \epsilon_i$?

2. (You may wish to revisit this question after the computing classes have started). The file `Problem_Set_05_Data.dta` contains 100 observations on the variables y , x_2 and x_3 . Consider the regression model

$$y_i = \beta_1 + \beta_2 x_{i2} + \epsilon_i, \quad i = 1, \dots, 100,$$

where interest centres on testing the null hypothesis $H_0 : \beta_2 = 1$ against the alternative $H_1 : \beta_2 \neq 1$.

Note: you may be interested to know that the data were generated as follows:

$$\begin{aligned} y_i &= \beta_1 + \beta_2 x_{i2} + \epsilon_i, & \epsilon_i &\sim NID(0, 1), & \beta_1 &= -1, & \beta_2 &= 1, \\ x_{i2} &= \alpha_1 + \alpha_2 x_{i3} + \epsilon_i, & \alpha_1 &= 0, & \alpha_2 &= 1, \end{aligned}$$

with x_{i3} distributed uniformly on the interval $[0, 15]$. Estimation of the model by OLS leads to the following results (please do this yourself after you are familiar with STATA):

y	Coef.	Std. Err.	t	P> t
x2	1.067901	.023261	45.91	0.000
_cons	-1.387372	.2009653	-6.90	0.000

- (a) Conduct a t -test of the hypothesis using the 5% significance level.
 (b) Is this test valid even if the errors are not normally distributed?
 (c) Suppose that x_2 might be correlated with ϵ . Write down the models that you would estimate by OLS in order to perform the IV regression of y on x_2 using x_3 as an instrument (please also estimate the two models after you are familiar with STATA).

- (d) Are the standard errors of the second stage OLS regression valid?
- (e) Estimation of the model by 2SLS using the appropriate STATA command leads to the following results (please also estimate the model after you are familiar with STATA):

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              y |          Coef.   Std. Err.      t    P>|t|
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              x2 |    1.011013    .0246807    40.96   0.000
              _cons |   -0.9647831   .2116257    -4.56   0.000
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Instrumented:  x2
Instruments:  x3
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How does the use of the IV estimator affect the inference drawn from a t -test of in part (a)?