University of Essex
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EC501 Econometric Methods and Applications

Problem Set 5: Sketch Solutions

Large Sample Methods (continued)

1. In this model, $X = [\ell,\ j]$ where ℓ is an $n \times 1$ vector of ones and $j = [1, 2, \ldots, n]'$. We already know (see Problem Set 1, Question 2(a), Model 4) that

$$X'X = \begin{bmatrix} n & \sum i \\ \sum i & \sum i^2 \end{bmatrix} = \begin{bmatrix} n & n(n+1)/2 \\ n(n+1)/2 & n(n+1)(2n+1)/6 \end{bmatrix}$$

and so

$$\frac{X'X}{n} = \begin{bmatrix} 1 & (n+1)/2 \\ (n+1)/2 & (n+1)(2n+1)/6 \end{bmatrix} \rightarrow \operatorname{plim} \frac{X'X}{n} = \begin{bmatrix} 1 & \infty \\ \infty & \infty \end{bmatrix}$$

and so the limiting matrix of $X'X/n$ is not finite.

2. (a) The t-statistic is $t = (1.0679 - 1)/0.0233 = 2.9142$ with a 5% critical value $t_{98}^{0.025} = 1.984$, therefore we reject the null against the alternative that $\beta_2 \neq 1$. An equivalent F statistic can be obtained with STATA.

   (b) If the errors were not normally distributed, the test would be asymptotically valid, and approximattely valid in large samples.

   (c) The two models would be as follows:

$$\begin{aligned} \text{First step:} \quad & x_{i2} = \alpha_1 + \alpha_2 x_{i3} + u_i, \\ \text{Second step:} \quad & y_i = \beta_1 + \beta_2 \hat{x}_{i2} + \varepsilon_i, \end{aligned}$$

   where $\hat{x}_{i2} = \alpha_1 + \alpha_2 x_{i3}$ are the fitted values from the first equation. The required STATA commands and the corresponding output are presented at the end of the file.

   (d) No, because σ^2 is estimated using the residuals $y - \hat{X} b_{IV}$ and not $y - X b_{IV}$.

   (e) The t-statistic is $t = (1.0110 - 1)/0.0247 = 0.4453$ with a 5% critical value $t_{98}^{0.025} = 1.984$, therefore we do not reject the null that $\beta_2 = 1$. This is in direct conflict with the inference made in part (a)!

Commands and output for part (c)

```
. * First stage
. reg  x2 x3

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,    98) = 1605.71
       Model |  1834.13498     1  1834.13498           Prob > F      =  0.0000
    Residual |   111.94157    98  1.14226091           R-squared     =  0.9425
-------------+------------------------------           Adj R-squared =  0.9419
       Total |  1946.07655    99  19.6573388           Root MSE      =  1.0688

------------------------------------------------------------------------------
          x2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x3 |   1.011136   .0252334    40.07   0.000     .9610609    1.061211
       _cons |   .0356089   .2132129     0.17   0.868    -.3875052     .458723
------------------------------------------------------------------------------

.
. * Save fitted values
. predict x2h, xb

.
. * Second stage
. reg y x2h

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,    98) =  410.32
       Model |  1874.75672     1  1874.75672           Prob > F      =  0.0000
    Residual |    447.7663    98  4.56904388           R-squared     =  0.8072
-------------+------------------------------           Adj R-squared =  0.8052
       Total |  2322.52302    99  23.4598285           Root MSE      =  2.1375

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         x2h |   1.011013   .0499111    20.26   0.000     .9119662     1.11006
       _cons |   -.964783   .4279646    -2.25   0.026    -1.814065   -.1155011
------------------------------------------------------------------------------
```