UNIVERSITY OF ESSEX
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DEPARTMENT OF ECONOMICS
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EC501 Econometric Methods and Applications

Problem Set 6

The Generalised Linear Regression Model and Heteroskedasticity

1. In the linear regression model $y_i = x_i'\beta + \epsilon_i$ $(i = 1, \ldots, n)$, x_i is a $K \times 1$ vector of regressors, β is an unknown $K \times 1$ vector of parameters, and $E(\epsilon_i|X) = 0$,

$$E(\epsilon_i \epsilon_j|X) = \begin{cases} \sigma_i^2, & i = j, \\ 0, & i \neq j. \end{cases} \quad (i = 1, \ldots, n).$$

   (a) In the above model, what are the properties of b, the ordinary least squares (OLS) estimator of β?

   (b) Explain how $var(b|X)$ can be estimated consistently.

   (c) Outline a method of testing for the presence of heteroskedasticity based on the OLS residuals $e_1, \ldots, e_n$, where $e_i = y_i - x_i'b$ $(i = 1, \ldots, n)$.

   (d) If $\sigma_1^2, \ldots, \sigma_n^2$ are known constants, explain how to obtain a best linear unbiased estimate of β.

   (e) If $\sigma_1^2, \ldots, \sigma_n^2$ are unknown functions of $x_i'\beta$, outline a method of obtaining a consistent estimate of β that may be more efficient than b, and explain how it can be used to conduct appropriate inference about β.

2. The Stata file `Problem_Set_06_Data.dta` contains 100 cross-sectional observations on household expenditure (y) and income (x) for a particular year. Using these data, estimate the model $y_i = \beta_1 + \beta_2 x_i + \epsilon_i$ by OLS.

   (a) Carry out a modified Breusch-Pagan LM test for heteroskedasticity using the variable x as the determinant of the variance of ϵ_i (use the command `estat hettest x, fstat`).

   (b) Now, perform the same test by explicitly estimating the auxiliary regression. Do your results match those obtained with the Stata command?

   (c) Obtain the heteroskedasticity robust standard errors for the OLS estimates of the parameters of this model.

   (d) Obtain FGLS estimates of the parameters of this model.

   (e) Conduct a test of $H_0 : \beta_2 = 0.9$ against $H_1 : \beta_2 \neq 0.9$ at the 5% level of significance using your preferred specification.

3. In the linear regression model $y = X\beta + \epsilon$, X is an $n \times K$ matrix of regressors, β is a $K \times 1$ vector of unknown parameters, $E(\epsilon|X) = 0$ and $E(\epsilon\epsilon'|X) = \sigma^2\Omega$.

   (a) Show that the GLS estimator, defined by $\hat{\beta} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$, satisfies $\hat{\beta} = \arg\min_\beta S_*(\beta)$ where $S_*(\beta) = (y - X\beta)'\Omega^{-1}(y - X\beta)$.

   (b) Assuming that $\lim_{n\to\infty} E\left[\left(\frac{X'\Omega^{-1}X}{n}\right)^{-1}\right] = \mathcal{Q}$ is a finite nonrandom matrix, show that $\hat{\beta}$ is a consistent estimator of β.

   (c) Assuming that $\text{plim}\,(X'\Omega^{-1}X)\,/n = Q_*$, $(1/\sqrt{n})(X'\Omega^{-1}\epsilon) \xrightarrow{d} N(0, \sigma^2 Q_*)$ as $n \to \infty$, derive the limiting distribution of $\sqrt{n}(\hat{\beta} - \beta)$.

   (d) Let $\hat{\beta}_F = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}y$ denote the FGLS estimator, where $\hat{\Omega}$ is a consistent estimator of Ω. Show that, if

$$\text{plim}\left[\left(\frac{X'\hat{\Omega}^{-1}X}{n}\right) - \left(\frac{X'\Omega^{-1}X}{n}\right)\right] = 0,$$

$$\text{plim}\left[\left(\frac{X'\hat{\Omega}^{-1}\epsilon}{\sqrt{n}}\right) - \left(\frac{X'\Omega^{-1}\epsilon}{\sqrt{n}}\right)\right] = 0,$$

   then $\sqrt{n}(\hat{\beta}_F - \beta)$ is asymptotically equivalent to $\sqrt{n}(\hat{\beta} - \beta)$ in the sense that

$$\text{plim}[\sqrt{n}(\hat{\beta}_F - \beta) - \sqrt{n}(\hat{\beta} - \beta)] = 0.$$