University of Essex
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EC501 Econometric Methods and Applications

Problem Set 7

Serial Correlation and Dynamic Models

1. Consider the first-order autoregressive, AR(1), process

$$\epsilon_t = \rho\epsilon_{t-1} + u_t, \quad |\rho| < 1,$$

where $E(u_t) = 0$, $var(u_t) = \sigma_u^2$ and $cov(u_t, u_s) = 0$ for $t \neq s$.

(a) Show that, for $s > 0$, ϵ_t has the representation

$$\epsilon_t = \rho^s\epsilon_{t-s} + \sum_{i=0}^{s-1}\rho^i u_{t-i}.$$

(b) Hence, or otherwise, derive $E(\epsilon_t)$, $\gamma_0 = var(\epsilon_t)$ and $\gamma_s = cov(\epsilon_t, \epsilon_{t-s})$ for $s > 0$.

(c) Derive the autocorrelations $\rho_s = \gamma_s/\gamma_0$ from the results in part (b). Calculate $\rho_0, \rho_1, \ldots, \rho_5$ when (i) $\rho = 0.1$ and (ii) $\rho = 0.9$. For which value of ρ is the process most correlated over time?

2. Consider the first-order moving average, MA(1), process

$$\epsilon_t = u_t + \lambda u_{t-1},$$

where $E(u_t) = 0$, $var(u_t) = \sigma_u^2$ and $cov(u_t, u_s) = 0$ for $t \neq s$.

(a) Derive $E(\epsilon_t)$, $\gamma_0 = var(\epsilon_t)$, $\gamma_1 = cov(\epsilon_t, \epsilon_{t-1})$ and $\gamma_s = cov(\epsilon_t, \epsilon_{t-s})$ for $s > 1$.

(b) Derive the autocorrelations $\rho_s = \gamma_s/\gamma_0$ from the results in part (a). Calculate $\rho_0, \rho_1, \ldots, \rho_5$ when (i) $\lambda = 0.5$ and (ii) $\lambda = 2$. What does this imply about the uniqueness of moving averages?

3. Consider the linear regression model

$$\begin{aligned} y_t &= x_t'\beta + \epsilon_t \qquad t = 1, \ldots, T, \\ \epsilon_t &= \rho\epsilon_{t-1} + u_t \quad |\rho| < 1, \end{aligned}$$

where x_t is a $K \times 1$ vector of observable regressors, β is an unknown $K \times 1$ vector of parameters, ρ is an unknown scalar, and u_t is an unobservable random disturbance with $E(u_t|X) = 0$, $E(u_t^2|X) = \sigma_u^2$ and $E(u_t u_s|X) = 0$ for $t \neq s$ (as before, X denotes the $n \times K$ matrix of regressors).

(a) What are the properties of the ordinary least squares (OLS) estimator, b, of β in the above model?

(b) Outline the LM test for the presence of first-order serial correlation based on the OLS residuals $e_t = y_t - x_t'b$ $(t = 1, \ldots, T)$.

(c) Show that y_t also has the representation

$$y_t = \gamma y_{t-1} + x_t'\alpha + x_{t-1}'\delta + u_t,$$

where γ is a scalar parameter and α and δ are $K \times 1$ vectors. How are γ, α and δ related to ρ and β? How would you estimate ρ and β using this model?

4. Stata file `Problem_Set_07_Data.dta` contains 200 time series observations on two variables, y and x_2, as well as an index t running from 1 to 200.

(a) Estimate the model $y_t = \beta_1 + \beta_2 x_{t2} + \epsilon_t$ and conduct a Breusch-Godfrey (LM) test for first-order serial correlation (perform the test both by using the appropriate Stata command and by estimating the corresponding auxiliary regression). What conclusions do you draw from this test?

(b) In the light of part (c) of Question 3, estimate the model

$$y_t = \mu + \gamma y_{t-1} + \alpha x_{t2} + \delta x_{t-1,2} + u_t$$

and carry out an LM test for the presence of first-order serial correlation. Is the test consistent with the tests in part (a)?

(c) In the light of part (c) of Question 3, carry out a Wald test of the restriction $\delta = -\gamma\alpha$.

(d) In view of the evidence obtained in parts (a)–(c), could y_t satisfy the model $y_t = \beta_1 + \beta_2 x_{t2} + \epsilon_t$ where $\epsilon_t = \rho\epsilon_{t-1} + u_t$? Explain.