

EC501 Econometric Methods and Applications

Problem Set 8

Unit Roots and Cointegration

1. Which of the following processes contain a unit root (where $\epsilon_t \sim WN(0, \sigma^2)$)?

- a) $y_t = 0.5y_{t-1} + \epsilon_t$; b) $y_t = 1.5y_{t-1} + \epsilon_t$;
c) $y_t = 1.0y_{t-1} - 0.25y_{t-2} + \epsilon_t$; d) $y_t = 1.5y_{t-1} - 0.50y_{t-2} + \epsilon_t$;
e) $y_t = 2.5y_{t-1} - 1.00y_{t-2} + \epsilon_t$; f) $y_t = 2.0y_{t-1} - 1.00y_{t-2} + \epsilon_t$.

2. Use the DF/ADF tests to determine which of the following time series might have unit roots? (The 5% critical value for the DF/ADF test in each case is -2.89 ; the 1% critical value is -3.51).

$$\widehat{\Delta x}_t = \begin{matrix} 1.80 & - & 0.65x_{t-1}, & R^2 = 0.99 \\ (0.29) & & (0.11) & \end{matrix}$$

$$\widehat{\Delta y}_t = \begin{matrix} 0.51 & + & 0.45y_{t-1} & + & 0.20\Delta y_{t-1}, & R^2 = 0.99 \\ (3.55) & & (0.09) & & (0.05) & \end{matrix}$$

$$\widehat{\Delta z}_t = \begin{matrix} 0.53 & - & 0.94z_{t-1} & + & 0.14\Delta z_{t-1} & - & 0.10\Delta z_{t-2}, & R^2 = 0.99 \\ (0.43) & & (0.30) & & (0.04) & & (0.04) & \end{matrix}$$

Figures in parentheses are standard errors.

3. Stata file `Problem_Set_08_Data.dta` contains 200 observations on five variables: u , a $N(0, 1)$ random variable; z , an AR(1) process with intercept 0.5 and slope 0.7; x , a random walk; w , a random walk with drift 0.05; and y , which is known to be I(1). In Stata, the `dfuller` command can be used to compute Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) statistics; type `help dfuller` for details of usage.

- (a) Plot the series u , z , x , and w , and decide whether or not it is appropriate to include the trend in the regression to perform the DF/ADF tests.
- (b) Compute the DF and ADF(1) statistics for u , z , x , and w , using the results of (a) to decide whether or not to include a trend in the regression (but maintaining a drift, or constant, term). What do these statistics suggest about the presence of unit roots at the 5% level of significance?
- (c) Do your results agree with what you know about the series?

- (d) Recall that both y and x are known to be $I(1)$.
- i. Estimate the regression of y on x and save the residuals.
 - ii. Test the hypothesis that the residuals are stationary (the appropriate 5% critical value for this test is -3.34). What do you conclude about the cointegration between y and x ?
 - iii. Estimate an error correction model of the form

$$\Delta y_t = \lambda e_{t-1} + \sum_{i=1}^2 \alpha_i \Delta y_{t-i} + \sum_{i=0}^2 \delta_i \Delta x_{t-i} + \epsilon_t,$$

and check for serial correlation in ϵ_t .