University of Essex
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EC501 Econometric Methods and Applications

Problem Set 8: Sketch Solutions

Unit Roots and Cointegration

1. The AR(1) processes in (a) and (b) are of the form $y_t = \gamma_1 y_{t-1} + \epsilon_t$, and we need to find the roots of the equation $\gamma(z) = 1 - \gamma_1 z = 0$, while for parts (c)–(f), the AR(2) processes are of the form $y_t = \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \epsilon_t$, and so we need the roots of the equation $\gamma(z) = 1 - \gamma_1 z - \gamma_2 z^2 = 0$. In all cases, if the roots lie outside the unit circle (modulus greater than one) the process is stationary; if their modulus is equal to one there is a unit root; while if they are inside the unit circle (modulus less than one) the process is nonstationary, or explosive. In fact, all the roots turn out to be real-valued. In the quadratic cases we can use the following formula for the roots of the equation $az^2 + bz + c = 0$:

$$z^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

   (a) The root of $(1 - 0.5z) = 0$ is equal to 2 and so y_t is stationary.

   (b) The root of $(1 - 1.5z) = 0$ is equal to 2/3 and so y_t is not stationary.

   (c) The roots of $(1 - z + 0.25z^2) = 0$ are of the form (with $a = 0.25$, $b = -1$ and $c = 1$):

$$z^* = \frac{1 \pm \sqrt{1-1}}{.5} = 2.$$

   Here there is a repeated root of 2 so that y_t is stationary.

   (d) The roots of $(1 - 1.5z + 0.5z^2) = 0$ are of the form (with $a = 0.5$, $b = -1.5$ and $c = 1$):

$$z^* = \frac{1.5 \pm \sqrt{2.25-2}}{1} = 1.5 \pm .5,$$

   so the roots are equal to 1 and 2. Here there is a root on the unit circle (equal to 1) so that y_t has a unit root. In fact, we can write $(1 - 1.5z + 0.5z^2) = (1 - z)(1 - 0.5z)$ and so y_t has the representation $\Delta y_t = 0.5\Delta y_{t-1} + \epsilon_t$ i.e. the first differences follow a stationary AR(1) process.

   (e) The roots of $(1 - 2.5z + z^2) = 0$ are of the form (with $a = 1$, $b = -2.5$ and $c = 1$):

$$z^* = \frac{2.5 \pm \sqrt{6.25-4}}{2} = 1.25 \pm 0.75.$$

   Here the roots are equal to 0.5 and 2, so that y_t is not stationary.

   (f) The roots of $(1 - 2z + z^2) = 0$ are of the form (with $a = 1$, $b = -2$ and $c = 1$):

$$z^* = \frac{2 \pm \sqrt{4-4}}{2} = 1.$$

   There is a repeated root of one, implying that y_t has two unit roots i.e. y_t is I(2). This is because $(1 - 2z + z^2) = (1 - z)^2$, we can write $\Delta^2 y_t = \epsilon_t$.

2. For x the DF statistic is $DF = -0.65/0.11 = -5.9091 < -3.51$; hence we reject the null hypothesis of a unit root at the 1% level of significance.
For y, the $ADF(1)$ statistic is $ADF(1) = 0.45/0.09 = 5 > -2.89$; hence we do not reject the null hypothesis of a unit root in y.
For z, the $ADF(2)$ statistic is $ADF(2) = -0.94/0.30 = -3.1333$, which lies between the two critical values. Hence we do not reject the null hypothesis of a unit root at the 1% level, but we do reject the null at the 5% level.

3. (a) From the plots, we see that only w has a clear trend. So, this is the only series for wich a trend should be included in the DF/ADF regressions.

   (b) The relevant 5% critical values for the DF/ADF statistics are -2.88 for regressions without trend, and -3.44 for regressions with trend. The results may be summarised as follows:

   u: Both statistics are less than the critical values and so we reject the null of a unit root.

   z: Both statistics are less than the critical values and so we reject the null of a unit root.

   x: Both statistics are above the critical value and so we are unable to reject the null of a unit root.

   w: Both statistics are above the critical value and so we are unable to reject the null of a unit root.

   (c) Yes, all the results agree with what we know about the series.

   (d) i. The estimation results are as follows

y	Coef.	Std. Err.	t	P>|t|
x	.8170007	.0165779	49.28	0.000
_cons	1.213493	.0870969	13.93	0.000

   ii. The Augmented Dickey-Fuller test statistic for unit root (computed with no constant and one lag) is -10.921, which is much smaller that the appropriate 5% critical value, which is -3.34. So, there is evidence that the residuals are stationary, which suggests that y and x are cointegrated.

iii. The estimation results and serial correlation test are as follows

D.y	Coef.	Std. Err.	t	P>|t|
e				
L1.	-1.23015	.1315104	-9.35	0.000
y				
LD.	.1463205	.1022398	1.43	0.154
L2D.	.0395507	.0710958	0.56	0.579
x				
D1.	.6885171	.0684117	10.06	0.000
LD.	-.2802397	.110956	-2.53	0.012
L2D.	-.1722079	.0949852	-1.81	0.071
_cons	-.003155	.0673453	-0.05	0.963

Breusch-Godfrey LM statistic: .8252708 Chi-sq(1) P-value = .3636