University of Essex — Department of Economics
Session: 2011/12 — João Santos Silva

EC501 Econometric Methods and Applications

Problem Set 9

Simultaneous Equations Models

1. In the model

$$y_1 = \gamma_1 y_2 + \beta_1 x_1 + \epsilon_1,$$

$$y_2 = \gamma_2 y_1 + \beta_2 x_2 + \epsilon_2,$$

y_1 and y_2 are $T \times 1$ vectors of observations on two endogenous random variables and x_1 and x_2 are $T \times 1$ vectors of observations on two exogenous variables. In addition, the ϵ_{ti} are unobservable random errors such that

$$E(\epsilon_{ti}|x_1, x_2) = 0, \quad i = 1, 2, \text{ and } t = 1, \ldots, T;$$

$$E(\epsilon_{ti}\epsilon_{sj}|x_1, x_2) = \begin{cases} \sigma_{ij} & \text{if } t = s \\ 0 & \text{if } t \neq s \end{cases}, \quad i, j = 1, 2, \text{ and } t, s = 1, \ldots, T.$$

   (a) State the order condition for identification. Use the order condition to suggest whether the parameters of the above model are identified.

   (b) Why is ordinary least squares an inappropriate method of estimation for the above model?

   (c) Write down the reduced form of the model.

   (d) Describe how you could estimate the model using two-stages least squares.

2. In the model

$$y_{t1} + \gamma_{21} y_{t2} + \beta_{11} x_{t1} + \beta_{21} x_{t2} + \beta_{31} x_{t3} = \epsilon_{t1},$$

$$\gamma_{12} y_{t1} + y_{t2} + \beta_{12} x_{t1} + \beta_{22} x_{t2} + \beta_{32} x_{t3} = \epsilon_{t2},$$

the y_{ti} are endogenous random variables, the x_{ti} are exogenous variables, and the ϵ_{ti} are unobservable random disturbances such that

$$E(\epsilon_{ti}|x_1, x_2, x_3) = 0, \quad i = 1, 2, \text{ and } t = 1, \ldots, T;$$

$$E(\epsilon_{ti}\epsilon_{sj}|x_1, x_2, x_3) = \begin{cases} \sigma_{ij} & \text{if } t = s \\ 0 & \text{if } t \neq s \end{cases}, \quad i, j = 1, 2, \text{ and } t, s = 1, \ldots, T.$$

   (a) Using the order condition:

      i. Study the identification of the two equations when $\beta_{11} = 0$ and $\beta_{21} = 0$;

      ii. Study the identification of the two equations when $\beta_{11} = 0$ and $\beta_{32} = 0$;

      iii. Study the identification of the two equations when $\beta_{21} = 0$ and $\beta_{22} = 0$;

   (b) Write the reduced form of the model when $\beta_{11} = \beta_{21} = 0$ and $\beta_{22} = \beta_{32} = 0$.

3. The file `Problem_Set_09_Data.dta` contains the data used by Lawrence Klein to estimate a model for the US economy.[1] A brief description of Klein's model can be found on page 357 of Greene's book. Here, we will focus on the following subset of equations

$$\begin{aligned} C_t &= \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + \alpha_2 (W_t^P + W_t^G) + \varepsilon_{1t} \\ I_t &= \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 K_{t-1} + \varepsilon_{2t} \\ W_t^P &= \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 A_t + \varepsilon_{3t}. \end{aligned}$$

The model contains three additional endogenous variables (X_t, P_t and K_t), and two other exogenous variables (T_t and G_t).

(a) Classify the variables in the system.

(b) Estimate the reduced form for P_t and test the joint significance of the exogenous and predetermined variables excluded from the structural equation for I_t. What do the results suggest?

(c) Save the fitted values for P_t and regress I_t on $\hat{P}_t$, P_{t-1} and K_{t-1}. How do you interpret these estimates? Are the reported standard errors useful?

(d) Use the appropriate command to compute the 2SLS estimates of the structural parameters in the equation for I_t. How do these results compare with the ones obtained before?

(e) Estimate the structural equation for I_t by OLS. Compare the results with those of the 2SLS.

[1] Klein L.R. (1950), *Economic Fluctuations in the United States 1921-1941*, Cowles Commission Monograph No. 11, New York, Wiley. The data is available here: `http://www.stern.nyu.edu/~wgreene/Text/Edition6/TableF13-1.txt`.