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EC501 Econometric Methods and Applications

Problem Set 10

Panel Data Models

1. Consider the fixed effects model for panel data given by

$$y_{it} = x_{it}'\beta + \alpha_i + \epsilon_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T,$$

where y_{it} is a scalar dependent variable, x_{it} is a $K \times 1$ vector of observable regressors which does *not* include an intercept, β is a $K \times 1$ vector of unknown parameters, $\alpha_1, \ldots, \alpha_n$ are unknown scalar parameters, and ϵ_{it} is a scalar random disturbance such that $E(\epsilon_{it}|x_{it}) = 0$, $E(\epsilon_{it}^2|x_{it}) = \sigma^2$ and $E(\epsilon_{it}\epsilon_{js}|x_{it}) = 0$ if $i \neq j$ or $t \neq s$.

   (a) Show that this model can be written

$$y = X\beta + D\alpha + \epsilon,$$

   where y is an $nT \times 1$ vector of observations on y, X is an $nT \times K$ matrix of observations on the x variables, $D = I_n \otimes 1_T$, 1_T is a $T \times 1$ vector of ones, $\alpha = [\alpha_1, \ldots, \alpha_n]'$, and ϵ is an $nT \times 1$ vector of random disturbances.

   (b) Show that the OLS estimators of β and α, denoted b and a respectively, satisfy

$$b = (X'M_D X)^{-1}X'M_D y \quad \text{and} \quad a = (D'D)^{-1}D'(y - Xb),$$

   where $M_D = I - D(D'D)^{-1}D'$.

   (c) Derive the covariance matrix of b.

2. The Stata file `Problem_Set_10_Data.dta` contains 15 yearly observations on cost data for six US airlines (the data are from Table F6-1 in Greene), as well as indices and airline-specific dummies, as follows:

i	airline index	t	time index	q	output
c	total cost of production	lf	load factor	pf	fuel price
	$d_1, \ldots, d_6$	airline-specific dummies			

Let $y = \ln(c)$ and $x' = [\ln(q),\ \ln(pf),\ lf]$.

   (a) Estimate the model $y_{it} = x_{it}'\beta + \alpha + \epsilon_{it}$ by OLS, where the ϵ_{it} are assumed to have the same properties as in Question 1.

   (b) Estimate the fixed effects, or least squares dummy variable model, given by $y_{it} = x_{it}'\beta + \alpha_i + \epsilon_{it}$, and carry out a test of the hypothesis that $\alpha_i = \alpha$ $(i = 1, \ldots, 6)$.

   (c) Estimate the random effects model $y_{it} = x_{it}'\beta + \alpha + u_i + \epsilon_{it}$ where ϵ_{it} is as above and $E(u_i) = 0$, $E(u_i^2|x_{it}) = \sigma_u^2$, $E(u_i u_j|x_{it}) = 0$ $(i \neq j)$ and $E(\epsilon_{it}u_j|x_{it}) = 0$ for all i, t and j. Carry out a test of the hypothesis that $\sigma_u^2 = 0$. What additional assumption is required for this estimator to be consistent?