Q1: Read "Analysis: Taxing the wealthy: Differences at the margins", FT, 24/10/2011, Monday (not in the CMR; the uni. does not subscribe to the paper in this year so that I cannot download it).

Q2: Find and read the latest 10-Q form of "Google", "Microsoft", and any other companies you think are very profitable. Find the leverage rate (namely the debt-to-equity ratio) of each of the them.

Q3: Show the irrelevance of the dividend policy (namely when to pay the dividend) to the value of the firm, when all the conditions of MM are satisfied.

A: Suppose that two firms have the same asset and are both all equitied. The difference is that firm 1 announces that it will distribute all the cash flow of this year, X_0, to the equity holders as dividend, whereas firm 2 announces that it will not distribute any dividend, but retain all the cash X_0 and invest it to the perfect capital market. Let the total value of firm 1's shares be $\pounds S_1$ after the announcement but before the dividend payment, and let the total value of firm 2's shares be $\pounds S_2$. We are going to prove that $S_1 = S_2$.

If $S_1 > S_2$, there is arbitrage as follows. A shareholder of firm 1 sells £1 of the firm's shares and buys £1 of firm 2's shares and meanwhile borrow $\frac{1}{S} \cdot X_0$ from the perfect capital market. By this transaction, in this year, he gives up dividend payment $\frac{1}{S_1} \cdot X_0$ from firm 1, gets nothing from firm 2 (because it does not pay dividend), but gets $\frac{1}{S_1} \cdot X_0$ from the borrowing, so that in net he breaks even; in any future date t, he gives up $\frac{1}{S_1} \cdot X_t$ from firm 1, gets $\frac{1}{S_2}(X_t + X_0 r)$ from firm 2 (where X_t is generated by the asset and $X_0 r$ by the investment of X_0 in the perfect capital market), and pays interest $\frac{1}{S_1} \cdot X_0 r$ for his borrowing. Then in net he gets: $-\frac{1}{S_1} \cdot X_t + \frac{1}{S_2}(X_t + X_0 r) - \frac{1}{S_1} \cdot X_0 r > 0$ if $S_1 > S_2$.

If $S_1 < S_2$, the arbitrage is done by selling £1 of firm 2's shares, buying £1 of firm 1's shares, and lending the dividend of firm 1, $\frac{1}{S_1} \cdot X_0$ from his shareholding to the perfect capital market.

Q4: In a two-period binomial economy, there are two firms that have identical cash flow: \$120 with probability one half and \$80 with probability one half tomorrow. Today Firm One issues 80 units of risk free debts and 20 shares, so each unit of debt returns \$1 tomorrow for certainty and each equity return \$2 in the good state and nothing in the bad one. Firm Two issues 100 shares, each of which returns \$1.2 in the good state and \$0.8 in the bad one.

There are two types of potential buyers of these securities. One kind is risk neutral, called deep pockets. The other type of investors, called households, is extremely risk averse, that is, they only care what they get at the worst case, for example, a firm 2's share is worth only \$0.8 to them. Overall the deep pockets have \$70 and households's capital is abundant. Assume the risk free rate, r_f, is 0.

(1): If deep pockets only invest with their own capital, that is, they have \$70 in total to make investment, what is the price of Firm One's debt, its equity and Firm Two's equity?

A: The price of the Firm One's debt, which is risk free is \$1 per unit, so it gives the same net return rate as the risk free asset. As some of Firm Two's shares are to be bought by households, its price is $p_2 = 0.8$. Then the gross return rate of the shares to deep pockets is $\frac{1}{p_2}$. It should equal the gross return of Firm One's equity to them, $\frac{1}{p_1}$ where p_1 denotes Firm One's share price. Therefore, $p_1 = p_2 = 0.8$.

(2): Are the values of the two firms equal? If they are not, how is MM's irrelevance theorem violated?

A: Firm One's debt is worth 80 and its equity is worth $20 \times p_1 = 16$. So Firm One's value is $V_1 = 80 + 16 = 96$. Firm Two's value is $V_2 = 100 \times p_2 = 80 < V_1$. the MM is violated because deep pockets are not borrowing.

(3): Suggested by the proof of MM's theorem, what shall happen then? Shall deep pockets borrow from households? What is, then, is the price of Firm One's debt, its equity and Firm Two's equity?

A: The proof suggests that deep pockets shall borrow from households to buy Firm Two's shares. Against each share, they can borrow \$0.8 which is the worst return of the share. Therefore, to buy one share of Firm Two, the deep pockets only need to take out $p_2 - 0.8$. This leveraged purchase gives him tomorrow $\$(1.2 - 0.8) = 0.4$ in the good state and $\$(0.8 - 0.8) = 0$ in the bad state, and therefore give a gross return rate of

$$\frac{0.5 \times 0.4 + 0.5 \times 0}{p_2 - 0.8} = \frac{0.2}{p_2 - 0.8}.$$

On the other hand, against one share of Firm One, deep pockets can borrow nothing from households. They have to pay the whole price with their own capita and the gross return rate is $\frac{1}{p_1}$.

In equilibrium, leveraged purchase of Firm Two's shares and purchase of Firm One's shares shall have the same gross return rate, that is,

$$\frac{0.2}{p_2 - 0.8} = \frac{1}{p_1}. \tag{1}$$

That is equivalent to

$$\begin{aligned} 100p_2 &= 20p_1 + 80 \Leftrightarrow \\ V_2 &= V_1, \end{aligned}$$

that is, the MM is restored.

The price of Firm One's debt, which is pinned down by the risk free return rate, is always 1. To find the two shareprices, we have already equation (1) above. To get another equation, note that the aggregate demand of the deep pockets' capital to buy these shares is: $100(p_2 - 0.8) + 20p_1 = 40p_1$. And the aggregate supply of their capital is \$70. If all this capital is used to buy the two firms' shares, then $40p_1 = 70 \Rightarrow p_1 = \frac{7}{4} > 1$, which is impossible. Therefore, not all the deep pockets' capital is used to buy the shares and some of it flows to buy risk free assets, which has gross return rate 1 to them. The investment in shares shall have the same return rate to the deep pockets as the investment in the risk free assets, that is,

$$\frac{1}{p_1} = 1.$$

Thus

$$p_1 = p_2 = 1.$$