

Question 1: Under the assumption $r_D = r_E = r_A$, derive that $V_2 = V_1 + V_D(1 - \frac{(1-t_c)(1-t_d)}{1-t_p})$, where t_c is the tax rate for corporate profit, t_d is that for dividend, and t_p is personal tax chargeable for interest income.

EBIT = C

$$V_1 = \frac{C(1-t_c)(1-t_d)}{r_A}$$

$V_2 = E + V_D$. Out of EBIT, $(C - I)(1 - t_c)(1 - t_d)$ goes to equity holders. Therefore,

$$E = \frac{(C - I)(1 - t_c)(1 - t_d)}{r_E}$$

And $I(1 - t_p)$ goes to debt holders. Therefore,

$$V_D = \frac{I(1 - t_p)}{r_D}$$

$$\begin{aligned} V_2 = E + V_D &= \frac{(C - I)(1 - t_c)(1 - t_d)}{r_E} + \frac{I(1 - t_p)}{r_D} = \frac{(C - I)(1 - t_c)(1 - t_d) + I(1 - t_p)}{r_A} \\ &= \frac{C(1 - t_c)(1 - t_d) + I[(1 - t_p) - (1 - t_c)(1 - t_d)]}{r_A} \\ &= \frac{C(1 - t_c)(1 - t_d)}{r_A} + \frac{I[(1 - t_p) - (1 - t_c)(1 - t_d)]}{r_A} \\ &= V_1 + \frac{V_D r_D}{(1 - t_p)} \frac{[(1 - t_p) - (1 - t_c)(1 - t_d)]}{r_A} \\ &= V_1 + V_D \left[1 - \frac{(1 - t_c)(1 - t_d)}{1 - t_p} \right] \end{aligned}$$

Question 2: Assume that Midco Industries wants to boost its stock price. The company currently has 20 million shares outstanding with a market price of \$15 per share and no debt. Midco has had consistently stable earnings, and pays a 35% tax rate. Management plans to borrow \$100 million on a permanent basis and they will use the borrowed funds to repurchase outstanding shares. After announcing the news of share repurchasing, what is the share price? How many shares can be repurchased?

A: For the former question, the gain for shareholders comes from the loss of old debt holders. Here the gain comes from the tax shield of debt.

Without Leverage

$$V^U = (20 \text{ million shares}) \times (\$15/\text{share}) = \$300 \text{ million}$$

If Midco borrows \$100 million using permanent debt, the tax shield of the debt is (note \$100 million is the market value of the debt, namely, the amount of cash the firm gets by issuing the debt):

$$PV(\text{Tax shield}) = tD = 35\% \times \$100 \text{ million} = \$35 \text{ million}$$

Thus the value of equity is now:

$$V^L = V^U + tD = \$300 \text{ million} + \$35 \text{ million} = \$335 \text{ million}$$

And after announcing the news of share repurchasing, share price will immediately jump

$$\$335 \text{ million} \div 20 \text{ million shares} = \$16.75.$$

The number of shares to be purchased is:

$$\frac{100}{16.75} = 5.97 \text{ million.}$$

Question 3: In a two-date economy, firm W's current asset expects to generate cash flow £100 million at the future date if the good state occurs and £15 million if the bad state occurs. Each state will occur with probability one half. Currently the firm has 10 million shares and no debt outstanding. The corporate tax rate is 20%. The firm now is considering issuing debt to repurchase 4 million shares. Suppose the potential debt holders and current shareholders are all risk neutral and the risk free rate is 0.

(a) What is the share price before the firm announces the deal of issuing debt to repurchase 4 million shares?

A: The value of the asset, which equals the value of the equity, is

$$(0.5 \cdot 100 + 0.5 \cdot 15) \times (1 - 20\%) = 46.$$

The share price is therefore $\frac{46}{10} = 4.6$.

(b) Will the share price change with the announcement?

A: Yes, because the firm then gets the tax shield benefit of debt.

(c) Find the market value and the face value of the debt.

A: Let the market value of the debt be D . Then the value of tax shield is $Dt = 0.2D$. The equity value increases by this amount and now equals $46 + 0.2D$. Thus the share price, after the announcement, is $\frac{46+0.2D}{10}$. The market value of the debt exactly suffice to buy 4 million shares, that is,

$$\frac{46 + 0.2D}{10} \times 4 = D.$$

Solve it, $D = 20$.

To find the face value, notice that $D > 15$, the bad state asset value. Therefore, the firm will default in the bad state. Let the face value be F . Then, $0.5 \cdot F + 0.5 \cdot 15 = D = 20$, which means $F = 25$.

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