

Question 1:

Consider a single period binomial setting where the risk free interest rate is zero (so the discount rate is 1) and everyone is risk neutral. A firm produces cash flows of \$ 180 if the economy is good and \$ 80 if the economy is bad. The good and bad states occur with equal probability. Initially, the firm has 100 shares outstanding and debt with a face value of \$100 due at the end of the period. The debt is protected by a covenant that it is senior to all newly issued debt.

The firm identifies a project that requires investment of \$47 and returns \$52 certainly. Assume that the project has to be financed with new debt issuance.

(ii) What is the face value of the newly issued debt? Show that the manager who acts in the interest of equity holders will not carry out the project.

(iii) Show that if the debt covenant is absent so that the new debt is as senior as the existing debt, then the manager will carry out the project. Are the holders of existing debt better off?

A for (ii): If the old debt is senior to the new debt (F is the face value of new debt):

	Asset	project	Old debt	New debt	R. to old debt	R. to new debt	Net to Equity
Good state (0.5)	\$180	\$52	\$100	\$F	\$100	\$F	\$132 - F
Bad state (0.5)	\$80	\$52	\$100	\$F	\$100	\$32	0

The equity value if not financing the project by issuing new debt:  $0.5 \cdot \$80 = \$40$ .

The equity value if financing the project by issuing new debt:

The face value of the new debt satisfies:

$$0.5 \cdot 32 + 0.5 \cdot F = 47$$

So  $F = 62$ .

Then if issuing the debt to finance the project, the equity value is  $0.5 \cdot \$ (132 - 62) = \$35$ . It is less than \$40, the equity value with the project not financed. Therefore, the manager will **NOT** carry out the project. ■

A for (ii): If the old debt is as senior as the new debt:

	Asset	project	Old debt	New debt	R. to old debt	R. to new debt	Net to Equity
Good state (0.5)	\$180	\$52	\$100	\$F	\$100	\$F	\$132 - F
Bad state (0.5)	\$80	\$52	\$100	\$F	$\frac{100}{100 + F} \cdot 132$	$\frac{F}{100 + F} \cdot 132$	0

The face value now satisfies:

$$0.5 \cdot \frac{F}{100 + F} \cdot 132 + 0.5 \cdot F = 47$$

So  $F = 50$ .

Then if issuing the debt to finance the project, the equity value is  $0.5 \cdot \$ (132 - 50) = \$41 > \$40$ , the equity value with the project not financed. Therefore, the manager will carry out the project.

The old debt holders are better off with the project financed: The net value of the project is \$5 and the equity gets \$1; therefore the old debt holders get \$4 more by giving up the debt covenant. You can also get this number by comparing what they get in the bad state (they get \$100 always in the good

state). Insisting on the debt covenant, they get \$80 in bad state, while giving it up they get  $\$ \frac{100}{100+50} \cdot 132 = \$88$ . ■

Question 2:

Suppose the manager of a firm identifies a project that requires investment of \$10 and returns \$15. The situation of the current asset and the debt outstanding is given by the following table.

	curr. asset R.	debt outstanding
good state (0.5)	\$20	-\$18
bad state (0.5)	£2	-\$18

Assume that the old debt is protected with the highest seniority and that the project has to be financed with new debt issuance.

(i) Show that the manager will not carry out the project.

A:

	Asset	project	Old debt	Net to E
Good state (0.5)	\$20	\$15	-\$18	\$17
Bad state (0.5)	\$2	\$15	-\$18	0

The value of the Equity if not carrying out the project:  $0.5 \cdot \$2 = \$1$ .

If the project is financed, the value of equity and the value of new debt is

$$V_{equity} + V_{new\ debt} = 0.5 \cdot \$17 + 0.5 \cdot 0 = \$8.5.$$

By limited liability,  $V_{equity} \geq 0$ . Therefore  $V_{new\ debt} \leq 8.5$ . But to finance the project, the market value of the new debt shall be 10. Therefore, the firm cannot finance the project by issuing new debt and thus has to give up the project.

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(ii): Suppose now the holders of existing debt are willing to lower the face value of their debt. What is the optimal face value to them?

A:

	Asset	project	Old debt	Net to E
Good state (0.5)	\$20	\$15	-\$ F	$35 - F$
Bad state (0.5)	\$2	\$15	-\$ F	$\max(0, 17 - F)$

When the face value of debt is now F, The value of equity without the project being financed is  $0.5 \cdot (20 - F)$ .

If the project is financed,  $V_{equity} + V_{new\ debt} = 0.5(\max(0, 17 - F) + 35 - F)$ .

To finance the project,  $V_{new\ debt} = 10$ .

Therefore, if the project is financed,  $V_{equity} = 0.5(\max(0, 17 - F) + 35 - F) - 10$ .

The debt holders benefit from lowering the face value only if that induces the equity to finance the project, that is, the equity value without the project is smaller than that with it, or

$$0.5 \cdot (20 - F) \leq 0.5(\max(0, 17 - F) + 35 - F) - 10$$

Equivalent to:

$$F \leq 12.$$

So if the debt holders want to lower the face value of the debt, the optimal new level is 12: if  $F > 12$ , the equity is still unwilling to finance the project and the lowering face value is not profitable to the debt holder; on the other hand,  $F < 12$  is not optimal, because at  $F = 12$ , the equity is already willing to finance the project.

At  $F = 12$ , the value of debt is \$12. If the debt holders insist on the face value of 18, the value of debt is  $0.5 \cdot \$18 + 0.5 \cdot \$2 = \$10$ . Therefore, they are willing to lower it to \$12.

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