

Q1 (Split Cash Flow): Consider the effects of splitting the cash flow $y(a)$ in the Aghion-Bolton model.

(i): Think, intuitively, what the effect is under E-control, and what it is under C-control.

(ii): Show that if $y(a^*) \geq K$, the optimal social value can be realized by properly splitting the cash flow under C-control, namely, control switch is not necessary.

(iii): Let $y(a) = 2\sqrt{A - a}$ and $B(a) = 2x\sqrt{a}$, where x measures the relative importance of the private benefit. Show that E-control is feasible with a proper way of splitting the cash flow if x is small enough and that if $x = 1$ and $K \geq \frac{\sqrt{A}}{2}$, no way of cash flow splitting can render E-control feasible.

Q2 (Control Switch and Debt): Suppose that in the Aghion-Bolton model, the cash flow depends on an exogenous random variable θ as follows, $y(a, \theta) = \alpha(\theta)z(a) + \beta(\theta)$, but the private benefit, $b(a)$, is independent of θ . θ is *contractible* and is realized after the contract is signed and before the action a is chosen. As in the lecture, $z'(a) < 0$, $b'(a) > 0$, and $z(a) > 0$. Moreover, $\alpha'(\theta) < 0$. Consider a scheme (namely a contract) in which the switch of control depends on the realization of θ .

(i): Think, intuitively, what features the optimal switch scheme should have.

(ii): Show that in the optimal scheme, there is a cut-off θ^* such that E has control when $\theta \geq \theta^*$. That is, E-control happens for high end part of θ .

(iii): Under which condition, the optimal scheme looks like a real life debt contract? Under which condition it does not?