# 1 Moral Hazard: Practice Questions

## 1.1 2x2x2 model, risk nuetrality, Limited liability

A principal wants to contract an agent to perform a task to produce output. There are two possible levels of output. The agent may put in high or low effort, but high effort is costly to the agent. Under high effort, there is a greater probability of high output (relative to under low output), but all outputs have a positive probability under any action. The principal does *not* observe the agent's action but *does* observe the output.

**Model:**

Principal (P) — employer

Agent (A) — worker

Agent chooses effort $e \in \{0, 1\}$

Agent receives transfer $t$

Cost or disutility of effort is $\psi(e)$ (Note: this assumes separability),

Let $A$'s utility be $U = t + \psi(e)$

Assume $\psi(0) = 0, \psi(e) = \psi$.

Assume the reservation wage is zero, and normalize reservation utility to 0, so $\bar{U} = u(0) = 0$

Output can either be high ($\bar{q}$) or low ($\underline{q}$) where $\bar{q} > \underline{q} > 0$

The probability of high output is higher if effort is high ( $e = 1$) than if it is low ($e = 0$), but both are positive.

I.e.,$\pi_1 > \pi_0 > 0$.

**(a)** Suppose first that both the principal and the agent are risk-neutral and have unlimited liability. **State the conditions that insure that the principal wants to hire the agent. State the conditions that insure that the principal would want to induce high effort if he could do so at the 'first-best' cost.** Assume both of these conditions hold. **Show that the first best outcome is attainable, irrespective of whether the principal observes the action of the agent or not, as long as the can principal observe (and can verify) the output.**

**Suggested Answer:** Principal's expected payoff is just the expected output minus wage: $E(q - t)$ $P$ wants to hire $A$ if $P$ can design a contract s.t. $E(q - t)$ $> 0$ is possible. He can induce low effort by offering a wage contract such that the worker gets $\psi(0) = 0$ for producing any output. This is worth

$$\pi_{low} = \pi_0 \bar{q} + (1 - \pi_0)\underline{q}$$

Since we assume positive probabilities and quantities this is preferred to shutting down.[1] *Result: If $P$ wants to induce high effort he must offer $A$ at least $t = \psi$. This yields:*

$$\pi^{FB} = \pi_1 \bar{q} + (1 - \pi_1)\underline{q} - \psi$$

[1](of course this is based on the normalization that $t_r$ is zero; otherwise we must assume that $\pi_{low} > t_r$)

*If $\Delta\pi(\bar{q}-\underline{q}) \geq \psi$ i.e., high effort is technically efficient, this is (perhaps weakly) preferred to inducing only low effort (which is preferred to shutting down).* However, since effort is not observed, we need to show that the first-best can be achieved here, i.e., participation and high effort can be induced without having to give the agent a rent. This will require a contract with one payment to high output: $\bar{t}$ and one to low output:$\underline{t}$ To induce participation and high effort two constraints must be satisfied: IC and PC

**Definition 1** *A contract satisfies the **Participation Constraint (PC)** if it ensures that the Agent(s) prefer(s) to take part in the contract rather than opt out.*

**Definition 2** *A contract satisfies the **Incentive Compatibility Constraint (IC)** if it ensures that the Agent(s) prefer(s) to take the action that the Principal wants them to take, rather than any other action.*

Here IC is $\pi_1\bar{t} + (1-\pi_1)\underline{t} - \psi \geq \pi_0\bar{t} + (1-\pi_0)\underline{t}$ i.e., $\Delta\pi(\bar{t}-\underline{t}) \geq \psi$ and PC is $\pi_1\bar{t} + (1-\pi_1)\underline{t} - \psi \geq 0$ Below, I show how to meet PC **at the first-best cost**, and then how IC can be met without imposing an additional cost.

**Theorem 3** *With a risk neutral agent and unlimited liability, there is no cost (relative to the perfect-information case) to providing incentives. I.e., the unobservability of effort is not costly.*

**Proof.** Choose $\underline{t}, \bar{t}$ so that PC is satisfied exactly:

$$EU_A = \pi_1\bar{t} + (1-\pi_1)\underline{t} - \psi = 0$$

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**Lemma 4** *There is no cost to increasing the difference between $\bar{t}$ and $\underline{t}$.*

**Proof.** If P raises $\bar{t}$ by $\frac{X}{\pi_1}$, he can lower $\underline{t}$ by $\frac{X}{1-\pi_1}$ and preserve the same utility level (and the IC). This will cost (in expectation) $\pi_1\frac{X}{\pi_1} - (1-\pi_1)\frac{X}{1-\pi_1} = X - X = 0$, i.e., will be costless. Thus the IC: $(\pi_1-\pi_0)[u(\bar{t}) - u(\underline{t}\ )] = \psi$ can be satisfied as in the 'first-best' case at no additional cost.

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Example – set IC and PC to hold with equality:
**PC:** $\pi_1\bar{t} + (1-\pi_1)\underline{t} - \psi = 0 \implies \underline{t} = \frac{\psi - \pi_1\bar{t}}{(1-\pi_1)}$
**IC:** $\Delta\pi(\bar{t}-\underline{t}) = \psi$
$\implies \bar{t} = \frac{\psi}{\Delta\pi} + \underline{t} = \frac{\psi}{\Delta\pi} + \frac{\psi-\pi_1\bar{t}}{(1-\pi_1)} = \frac{\psi}{\Delta\pi}(1+\Delta\pi-\pi_1) = \frac{\psi}{\Delta\pi}(1-\pi_0)$
$\implies \underline{t} = \frac{\psi - \pi_1(\frac{\psi}{\pi_1-\pi_0}(1-\pi_0))}{(1-\pi_1)} = \psi\frac{\pi_0}{\pi_0-\pi_1}$ Note this latter wage is negative. Since PC holds with equality $P$ can do no better.[2]

**(b)** Maintain the assumptions of part (a), except that now the agent has limited liability: in specific, the principal can never offer the agent a wage lower

[2]Doublechecking $\pi_1\frac{\psi}{\pi_1-\pi_0}(1-\pi_0) + (1-\pi_1)\psi\frac{\pi_0}{\pi_0-\pi_1} - \psi = 0$

than her (the agent's) reservation wage, i.e., the maximum liability ($l$) is zero. Assume that under the first-best case (part a), the principal preferred to hire the agent and induce high effort.

**What contract could the principal propose to induce high effort? What would be the surplus of the principal and agent? Under what conditions will the principal want to induce positive effort? Under what conditions does the limited liability cause a loss of social surplus? Explain, demonstrating any claims you make.**

**Suggested Answer:** *In general:* Limited liability imposes two additional constraints: $\bar{t} \geq 0, \underline{t} \geq 0$. Of course, since $\bar{t} > \underline{t}$, $\underline{t}$, only the latter is required. In fact, this constraint will make the PC superfluous: limited liability (at zero) implies that the payment is never negative. If payment is never negative it must have a non-negative expected value, since 0 effort was costless and the reservation wage was 0, the agent (at least weakly) prefers to participate and not put in effort over not signing the contract. But the IC will ensure that the agent who participates prefers to put in high effort. So, the agent must prefer to participate and put in high effort over not participating – satisfying the PC automatically. Generally:

- If binding (which they will be here), these limited liability constraints require that $A$ gets a 'limited liability' rent if $P$ wants to induce effort.
- Since he can't impose the 'stick' (punishment) he must offer the 'carrot' (reward) to have a sufficient difference between $\underline{t}^{SB}$ and $\bar{t}^{SB}$
- Of course this additional cost of imposing effort means we require a higher social surplus from effort for $A$ to want to induce effort. If this yields that $P$ no longer wants to induce effort (which was technically efficient) limited liability has yielded a loss of total surplus. However, if actions are unchanged, the transfer of 'rent' from $P$ to $A$ has no impact on total social surplus.

*In this case:*
P's problem:

$$\max_{\bar{t},\underline{t}} \pi_1(\bar{q} - \bar{t}) + (1 - \pi_1)(\underline{q} - \underline{t})$$

$$\text{s.t. } \underline{LL} : \underline{t} \geq 0$$

$$IC : \Delta\pi(\bar{t} - \underline{t}) - \psi \geq 0$$

$$\Longrightarrow \bar{t} \geq \frac{\psi}{\Delta\pi} + \underline{t}$$

We know these constraints will hold with equality: If $\underline{t} > 0$, could lower $\underline{t}$ and $\bar{t}$ by the same 'small' amount, preserving IC (and $\underline{LL}$ of course, since it was slack), and increasing expected profit. If $\Delta\pi(\bar{t} - \underline{t}) - \psi > 0$, could lower $\bar{t}$ by a 'small' amount, preserving $\underline{LL}$ (and IC of course, since it was slack), and increasing expected profit. *Thus, if $P$ wants to hire $A$ and induce high effort $\bar{t}$ and $\underline{t}$ are defined by these* *constraints:*

$$\underline{t}^{SB} = 0$$
$$\bar{t}^{SB} = \frac{\psi}{\Delta\pi} + \underline{t} = \frac{\psi}{\Delta\pi}$$

This will yield an expected profit:

$$\pi^{SB} = \pi_1(\bar{q} - \frac{\psi}{\Delta\pi}) + (1-\pi_1)(\underline{q})$$

$P$ will compare this to his 'low-effort' profit:

$$\pi_{low} = \pi_0\bar{q} + (1-\pi_0)\underline{q}$$

**Comparing, inducing high effort is better if:**

$$\pi_1(\bar{q} - \frac{\psi}{\Delta\pi}) + (1-\pi_1)(\underline{q}) \geq \pi_0\bar{q} + (1-\pi_0)\underline{q}$$
$$\Longrightarrow \pi_1\bar{q} - \pi_0\bar{q} + (1-\pi_1)(\underline{q}) - (1-\pi_0)\underline{q} \geq \pi_1\frac{\psi}{\Delta\pi}$$
$$\Longrightarrow \Delta\pi(\bar{q} - \underline{q}) \geq \pi_1\frac{\psi}{\Delta\pi}$$

*Thus, if* $\Delta\pi(\bar{q} - \underline{q}) \geq \frac{\pi_1\psi}{\pi_1-\pi_0} = \frac{\pi_1\psi-\pi_0\psi+\pi_0\psi}{\pi_1-\pi_0} = \frac{(\pi_1-\pi_0)\psi+\pi_0\psi}{(\pi_1-\pi_0)} = \psi + \frac{\pi_0\psi}{\Delta\pi}$ *high effort is (perhaps weakly) preferred to inducing only low effort (which is preferred to shutting down). If* $\Delta\pi(\bar{q} - \underline{q}) \geq \psi$ *but* $\Delta\pi(\bar{q} - \underline{q}) < \psi + \frac{\pi_0\psi}{\Delta\pi}$ *then the agent will induce high effort under unlimited liability but *not* under limited liability. If this holds, limited liability creates an inefficiency. If this does not hold, limited liability has no impact on efficiency. If a 'rent' is passed to the agent, this is not a welfare loss.*

**(c) Suppose next that the principal is risk-neutral while the agent is risk-averse.**

**Show that if the principal does not observe the agent's action but does observe the output (and all outputs have a positive probability under any action), then the optimal contract cannot attain the first best outcome.**

**Suggested answer:**

See earliest lecture notes, dealing with this using various setups (two types of effort and output, many types of output, continuous effort...)

also in Laffont and Martimort.

Intuitively, the main reason the contract cannot achieve the first best outcome is that to give an incentive for effort (the $IC$ constraint), pay needs to be conditioned on output. But, since output involves a random component, this shifts the risk to the agent. The Agent must be compensated for this risk, which is a cost to the Principal, and in some cases will make it no longer worth inducing high effort. By shifting risk to the Agent, who is risk-averse, an inefficiency is generated relative to the perfect-information case, in which all risk was bourne by the Principal, who was risk-nuetral. But if this proves too costly, and the optimal effort is not induced, this is also less-then first-best, yielding lower total surplus and lower profit for the Principal, relative to when output is observable.

Note: this question may have been ambiguous, sorry. I wanted you to show that the optimal 'second best' contract will yield a lower profit for the principal, and a lower total surplus, than when effort is observed. You could show this by doing a model as in part (a) but where the agents utility $U_a(t)$ is a concave function, hence risk aversion.

For "first-best" we must have optimal risk-sharing, i.e., the risk-nuetral party, here the agent must bear all risk. But if he does not condition the agent's pay on output, the agent will surely put in low effort, since effort his costly. And if he conditions the agent's pay on output the agent is bearing some risk, since output is stochastic – even high effort may yield low output (and vice versa). Since, if he wants to induce high effort, he will have to compensate the agent for bearing this risk, total surplus is reduced. Since the principal captures all the surplus here (in either case), this means the principal's profit is reduced. The alternative is not to induce high effort; but if we assume, as we normally do, that effort is technically efficient, i.e., $(\pi_1 - \pi_0)(\bar{q} - \underline{q})$ $> \psi$, then not inducing high effort also involves a loss of total surplus and hence a loss of profit for the principal.

We know that the "first-best" contract will not work literally (since effort cannot be conditioned on), and if we were to write a contract that yielded the same expected transfer to the agent as under first-best (i.e., $\pi_1 \bar{t} + (1 - \pi_1)\underline{t} = \bar{t}^{fb} = \psi + \bar{U}$ this would not satisfy the agent's PC constraint, nor would ; the agent would get a negative surplus (in expectation) if she accepted such a contract, because, while the expected value of the transfer is the same as under first-best, there is now risk, and the agent is risk averse. More formally:

PC holds with equality $\Longrightarrow$

$$\pi_1 u(\bar{t}) + (1 - \pi_1) u(\underline{t}) = \psi + \bar{U}$$

From this we can prove that the expectation of the (minimum) transfer that induces high effort is higher under asymmetric information then under observable effort, i.e.,

$$\pi_1 \bar{t}^{sb} + (1 - \pi_1)\underline{t}^{sb} > t^* = \psi + \bar{U} = \psi$$

...because from strict concavity of $u(t)$, we have

$$\pi_1 u(\bar{t}) + (1 - \pi_1) u(\underline{t}) < u\left(\pi_1 \bar{t} + (1 - \pi_1)\underline{t}\right)$$

whenever $\bar{t} \neq \underline{t}$. Thus, here, if $\pi_1 \bar{t} + (1 - \pi_1)\underline{t} = \bar{t}^{fb} = \psi + \bar{U}$ we know that $\pi_1 u(\bar{t}) + (1 - \pi_1) u(\underline{t}) < \psi + \bar{U}$ and PC would not be satisfied.

## 1.2 Moral hazard in the principal-agent setting.

A principal ($P$) wants to contract an agent ($A$) to perform a task to produce output. Output can either be high ($\bar{q}$) or low ($\underline{q}$) where $\bar{q} > \underline{q} > 0$. The agent may put in effort $e \in \{0, 1\}$. Let $A$'s utility be $U = v(t) + \psi(e)$. Assume $\psi(0) = 0$, and $\psi(1) = \psi$. The probability of high output given $e = 1$ is $\pi_1$, and the probability of high output given $e = 0$ is $\pi_0$, where $\pi_1 > \pi_0 > 0$. Assume the reservation wage is $t_r$, and normalise reservation utility to 0, so $\bar{U} = u(t_r) = 0$. Assume the principal is risk-neutral.

**(a)** What assumption(s) over $v(t)$ ensure(s) that the agent is risk-averse?

$v''(t) < 0$, i.e., diminishing returns/concave utility should be sufficient to ensure risk aversion. With such preferences the agent will always prefer a certain outcome over an uncertain outcome with the same expected transfer.

**(b)** Suppose that effort is observable. Given this, solve for:

i. The conditions that ensure the principal wants to hire the agent and

ii. The conditions that ensure the principal will want to write a contract that ensures the agent puts in high effort.

Justify your answer.

i. The conditions that ensure the principal wants to hire the agent:

Since P can observe effort, he only needs to reimburse A for her effort costs and her reservation wage.

To induce high effort the principal needs to pay agent $t^{FB}$ high enough such that $v(t^{FB}) - \psi > u(t_r) = 0$, but the principal neither wants to nor needs to pay the agent any more than this. Hence the wage is $v(t^{FB}) = \psi(1) = \psi \Longrightarrow t^{FB} = v^{-1}(\psi) \equiv h(\psi)$ if he wants to induce high effort and $v(t^{FB}) = \psi(0) = 0 \Longrightarrow t^{FB} = v^{-1}(0) = h(0) = t_r$ if he does not want to induce high effort.

Thus, principal will hire the agent if $\pi_1\bar{q}+(1-\pi_1)\underline{q} \geq h(\psi)$ or if $\pi_0\bar{q}+(1-\pi_0)\underline{q} \geq t_r$.

ii. The conditions that ensure the principal will want to write a contract that ensures the agent puts in high effort:

We need $\pi_1\bar{q} + (1 - \pi_1)\underline{q} \geq h(\psi)$, so inducing high effort is better than nothing, as well as $\pi_1\bar{q} + (1 - \pi_1)\underline{q} - h(\psi) \geq \pi_0 v(\bar{q}) + (1 - \pi_0)v(\underline{q}) - t_r$, so inducing high effort is more profitable than inducing low effort.

Another way to state this second condition is:

$\Delta\pi\Delta q > h(\psi) - t_r$ , where $\Delta\pi = \pi_1 - \pi_0$ and $\Delta q = \bar{q} - \underline{q}$. This formulation can be interpreted as a "technical efficiency" condition.

**(c)** Now assume effort is unobservable, and assume the agent is risk averse. Given these assumptions, when will the principal prefer to induce high effort (explain using the concept of 'risk premium')? If the principal wants to induce high effort, describe (algebraically and intuitively) the contract he will choose. *Note: you do not need to give a proof of which constraints bind and which are slack, you can merely cite a general rule.*

To induce high effort without observing effort, P will have to make payments contingent on output, and meet an IC constraint. The payments for high and low output must be distinct enough that the benefit to the agent from putting in high effort (in terms of higher expected output and thus higher expected utility) must exceed the cost of putting in this effort. Since the agent is risk averse and output is stochastic, making payments contingent on output will require paying a 'risk premium' to compensate A for taking on this risk. In general, if the required risk premium is no greater than the net benefit of high effort (higher expected output minus effort costs), P will write a contract to induce high effort.

A risk premium is the additional amount over and above the expected value of a lottery that it is necessary to pay someone someone to be neutral between a lottery and its expected value.

Lets say the pay is $\bar{t}$ for high output and $\underline{t}$ for low output. The IC constraint and the PC will both bind, i.e., hold exactly for an optimal second-best contract, as we proved in lecture.

Hence $\pi_1 u(\bar{t})+(1-\pi_1)u(\underline{t}) = \psi$ [PC] (note $u(t_r) = 0$ by our normalizing assumption)
and $\Delta\pi(u(\bar{t}) - u(\underline{t}) = \psi$ [IC].
Hence $\bar{u} = \frac{\psi}{\Delta\pi} + \underline{u}$. Where $\underline{u} = u(\underline{t})$ and $\bar{u} = u(\bar{t})$
Plugging this to PC:
$\pi_1(\frac{\psi}{\Delta\pi}+\underline{u}) + (1-\pi_1)\underline{u}= \psi$
$\pi_1(\frac{\psi}{\Delta\pi})+\underline{u}= \psi$
$\underline{u}= \psi(1-\frac{\pi_1}{\pi_1-\pi_0}) = \psi-\frac{\pi_1\psi}{\pi_1-\pi_0}= -\psi\frac{\pi_0}{\pi_1-\pi_0}$
$\Longrightarrow \underline{t} = h(\psi(1-\frac{\pi_1}{\pi_1-\pi_0}))$ where $h()$ is the inverse utility function.
And $\bar{u} = \frac{\psi}{\Delta\pi}+\underline{u} = \frac{\psi}{\pi_1-\pi_0}-\psi\frac{\pi_0}{\pi_1-\pi_0} = \psi\frac{1-\pi_0}{\pi_1-\pi_0}-(\psi+\frac{(1-\pi_1)\psi}{\pi_1-\pi_0}) = \frac{\psi}{\pi_1-\pi_0}(1-\pi_0) = \psi + \frac{\psi(1-\pi_1)}{\pi_1-\pi_0}$
$\Longrightarrow \bar{t} = h(\psi\frac{1-\pi_0}{\pi_1-\pi_0})$

Thus, the contract $\bar{t} = h(\psi\frac{1-\pi_0}{\pi_1-\pi_0}),\underline{t} = h(\psi(1-\frac{\pi_1}{\pi_1-\pi_0}))$ will induce participation and high effort.

The contract $t = t_r$ (for high or low output) will induce participation and low effort.

If $\pi_1\bar{t} + (1-\pi_1)\underline{t} - t_r \leq \Delta\pi\Delta q$ (for the $\bar{t}$ and $\underline{t}$) specified above then the costs (including risk premium and compensation for effort) of inducing high effort are less than the benefit, so P will induce high effort even when he can not observe effort.

If we denote $R$ as the "risk premium" necessary to induce effort in such a case, we can say that the principal will
induce effort when $\Delta\pi\Delta q \geq h(\psi) + R$

**(d)** Compare the case where effort is observable (as in part b) to the case where effort is unobservable (as in part c). Will the total surplus (the sum of the the principal and the agent's surpluses) be as high when effort is unobservable as when effort is observable? Why or why not? Does the observability of effort affect the agent's surplus? Why or why not?

Define $C^{FB} = h(\psi)$ as the cost of inducing high effort when effort is observable, the "first-best" case. Define $C^{SB} = \pi_1 \bar{t} + (1-\pi_1)\underline{t} - t_r = h(\psi) + R$ as the cost of inducing high effort when effort is not observable, where $R$ denotes the "risk premium" necessary to induce effort in such a case.

If $\Delta\pi\Delta q < h(\psi) = C^{FB}$ then P will never induce high effort and the observability of effort does not matter; all surpluses are the same.

If $C^{FB} \leq \Delta\pi\Delta q < C^{SB}$, i.e., $h(\psi) \leq \Delta\pi\Delta q < h(\psi) + R$, then the unobservable effort implies an agency cost that is so high that the principal no longer wants to induce high effort, even though it is technically efficient. The necessary risk premium is too high to make it worth it. This implies lower total surplus, lower by $\Delta\pi\Delta q - h(\psi)$ than in the first-best case. The agent recieved no surplus in either case.

Finally, if If $C^{FB} < C^{SB} \leq \Delta\pi\Delta q$, i.e., $h(\psi) < h(\psi) + R \leq \Delta\pi\Delta q$ then P will induce high effort whether or not effort is directly observable. However, when it is unobservable, the contract shifts risk from the principal to the agent, which is inefficient, since the agent is risk-averse. The risk premium $R$ is a loss to total surplus; it is paid by P but, relative to the certain payment, is not a net gain to A. The agent still gets no surplus.

## 1.3 Moral hazard and limited liability in the principal-agent setting, multiple outputs

Consider the standard 'hidden action/moral hazard' setup, with a single principal and a single agent, two levels of effort and two levels of output. Both parties are risk-neutral.

**(a)** In this context, explain the concept of a 'limited liability rent.' When will the principal set a contract such that the agent gets a limited liability rent? Why does this occur? If the agent receives a limited liability rent, does this imply that the total surplus (the sum of the principal's and the agent's utility) is lower than the total surplus in the full information case? Explain the logic behind your answer.

If effort is unobservable, in order to induce high effort, the Principal must drive a wedge between the payoffs to low and high output. With risk nuetrality and unlimited liability the principal can "center" the payoffs at zero, i.e., have a zero expected payoff. However, if the lower payoff has high enough a floor, the expected payoff will be positive, hence earn the agent a "rent" (noting, since she is risk-neutral, any positive payoff essentially implies positive utility).

If an agent receives a LL rent this does not imply a lower total surplus. P only pays a LL rent (in the 2x2x2 case) if inducing high effort. If there is high effort, the production process is technically eficient, i.e., maximizes production minus effort costs. And, since the agent and principal are both risk-nuetral, the sharing of risk is inconsequential. The LL is a cost to P, but an equivalent gain to A, so the total surplus is unchanged.

**(b)** Now consider the 'hidden action/moral hazard' setup, with a single principal and a single agent and two levels of effort. But now suppose there are *three* possible levels of output. Suppose as well that the principal is risk neutral and the agent is risk-averse. The agent's liability is unconstrained. In this context, will a contract always offer a higher wage for a higher level of output? If so, explain why. If not, give a counter-example as well as an intuitive explanation.

The answer is no, a contract need not always offer a higher wage for higher effort. If the MLRP holds, the contract will offer a higher wage for a higher level of output. But it need not hold; it is a strong assumption. For a derivation and counterexample, see notes from lecture 2. See also part C of the next practice question.

**(c)** Consider again the 'hidden action/moral hazard' setup with a single principal and a single agent, two levels of effort, and *three* possible levels of output. State the 'maximum likelihood ratio criterion' (i.e., 'bang-bang' payments) and give a derivation of this condition or intuition for it. Explain why this holds for the case where the agent is risk neutral and has a binding limited liability constraint but not for the case where the agent is risk-averse.

The MLR criterion is that the contract only gives a strictly positive transfer for the level of output ($j$) that maximizes the likelihood ratio $\frac{\pi_{j1}-\pi_{j0}}{\pi_{j1}}$. Intuition is that the expected cost to increase a payment to an output $j$ is proportional to the probability of it occuring under high output, the denominator $\pi_{j1}$. The 'incentive benefit' of this payment is the extent to which it increases an agents expected payoff under high output relative to under low output $\pi_{j1} - \pi_{j0}$. If the agent is risk-nuetral, P will only reward the state that yields the highest incentive benefit per pound of expected cost. However, if agents are risk-averse, the marginal incentive benefit decreases in the payment to a particular output state, so it is better to spread the payment across states so as to not have to pay as high a risk premium.

## 1.4 Moral hazard in teams, general results on signals of effort and pay structure with several possible outputs

### a. Explain the problem of 'free-riding in teams' and when and how these can be resolved. Under what circumstances will teams find it useful to have a 'principal' who is not a member of the team?

**Suggested Answer:** *Main points:*

When output is produced in a team, and only the team's output is observed, there is a potential incentive to 'free ride' This 'public goods problem' can be 'solved' if we can implement a contract that involves an injection or extraction of surplus (reward/punishment larger than actual profit or loss). Assuming the members of a team cannot commit to throwing away output, this can only be accomplished with an external principal.

*Free rider problem : intuition*

Say $n$ of us are collaborating to produce research reports. The boss pays our team based on the number of good reports we produce, for which we bill the client $\pounds 1$ per report. We work on all the reports together, so there is no way to separate our output. She says 'you all will get the same amount, $\frac{\pounds 1}{n}$ for each report, and I want you all to put in the same effort.' She doesn't want to get involved in disputes: we have to work it out between ourselves. My effort $(e_i)$ is costly, increasingly costly as I work additional hours – say it costs me $v_i(e_i)$ (a concave function). Assume (WLOG) that my productivity remains constant at $x$ per hour. If I were producing these by myself I'd work until the marginal effort required to produce a report equalled my pay for the last report, and then stop. I.e., until marginal cost equals marginal benefit, $v_i'(e) = 1$. From concavity $v_i(e_i) < 1$ at this point, so I would get a surplus. This is technically efficient. Let's say the resulting output is $q^* = 1$ per person who works efficiently. But, in the team, with the incentives above, I only get $\frac{1}{n}$ of the pay from any report I write. So I will only work until $v_i'(e_i) = \frac{1}{n}$. I will 'slack off'. This is the classic 'free rider' problem in public goods models. Because of this problem, we the employees are worse off. This is not Pareto-efficient: if any of us could work and keep all of the gains he could make himself better off and all the others would be the same.

*Mechanism for free rider problem: intuition*

Unless we can observe each individual's effort and reward/punish him based on it (which may be difficult to enforce), as a team we can not solve this problem between ourselves. If everyone is to be paid the same, we need to increase every team members incentive by a factor of $n$ so that $v_i'(e_i) = n\frac{1}{n} = 1$. But there is no way to do this unless: if we *do* perform efficiently we produce $nq^* = n$ units and are paid $\pounds nq^* = \pounds n$ in total. But to increase incentives enough we'd have to pay everyone $\pounds n\frac{1}{n} = \pounds 1$ per unit of our total production, which would cost us $\pounds n * n$ – where would we get the money to pay ourselves this? If we could commit to us all having to throw away $\pounds\frac{n-1}{n}$ if we produce less than $n$ reports we would also have this efficient incentive – but commiting to throw away money may not be feasible!

However, the boss could implement these incentives! She could dock all of our pay by $\pounds\frac{n-1}{n}$ (i.e., pay us nothing) if we fail to produce the $n$ total reports! Under such a payment scheme, choosing efficient actions is a Nash equilibrium: if all others agents are putting in the efficient effort, I get $\pounds 1$ if I puts in optimal effort, and thus I makes the surplus $\pounds 1 - v(e^*)$ (assuming a linear separable utility function) as in the case where I worked alone, and $0$ otherwise. One caveat to this is that another Nash equilibrium is for no worker to put in any effort: if others are not putting in optimal effort, my best response is to put in none, since it yields me no gain. Another equilibrium may occur where some workers put in suboptimal effort and others put in $e > e^*$ to 'make up' for the suboptimal efforts of others. In a sense this contract is 'risky' if we are not sure which equilibrium will arise.

**b. In general, when will a principal (manager) find it optimal to**

**make an agent (worker's) pay a function not only of output but of 'signals' of effort? Explain your answer.**

**Suggested Answer:** With uncertainty and risk aversion (or need for budget balance), inducing optimal effort requires 'monitoring' of individual performance, and payments based on a 'sufficient statistic.' If the principal does not observe effort she might want to rely on such signals. Holstrom's informativeness principle notes that the principal varies wages between states with same output, different signal *if and only if* the signal yields some information about the action chosen. Stating this formally:. ...[*S*hould put more in an exam answer .... Explain why conditioning on an uninformative signal will be harmful with a risk-averse agent. ]

**c.** Now consider a case with only one principal and one agent, two types of effort ($\bar{e}$ and $\underline{e}$), and several levels of output $(q_1, q_2, ...q_n)$. The principal observes output but not effort. Assume effort impacts expected output in some way, but output also has a random component. Assume that $e = \bar{e}$ maximizes the social surplus (expected output minus effort). Let $\pi_{i1}$ be the probability of output $i$ under high effort, and $\pi_{i0}$ be the probability of output $i$ under high effort.

**Will the optimal 'second-best' contract always have pay nondecreasing in output? Prove this holds or give an example where the pay decreases as output increases over some range of output, and motivate this example. Discuss the difficulties that can ensue if pay decreases over some portion of output.**

**Suggested Answer:** For wages to be nondecreasing in output over all ranges, we require that Monotone Likelihood Ratio Property (MLRP): the likelihood ratio of output under the different levels of effort must be nondecreasing with respect to outputs. That is, the likelihood ratio $\frac{\pi_{i1}-\pi_{i0}}{\pi_{i1}}$, must not decrease as $i$ increases (indicating higher output). *Example where MLRP fails:*

$$\pi_{10} = \frac{1}{6}, \pi_{20} = \frac{2}{3}, \pi_{30} = \frac{1}{6},$$
$$\pi_{11} = \pi_{21} = \pi_{31} = \frac{1}{3}$$
$$\frac{\pi_{11} - \pi_{10}}{\pi_{11}} = \frac{1}{2}$$
$$\frac{\pi_{21} - \pi_{20}}{\pi_{21}} = -1$$
$$\frac{\pi_{31} - \pi_{30}}{\pi_{31}} = \frac{1}{2}$$

This is not monotone... so MLRP fails.

If $S_3$ is much larger than $S_1$ and $S_2$ then $P$ wants to induce $e = 1$. $q_2$ is more likely under a low than a high effort (and $q_1$ and $q_3$ are equally 'informative' of a high effort) Thus, $t_2 < t_1 = t_3$ to induce $e = 1$. Not monotonic! This could be interpreted as a case in which the agent's optimal effort involves a huge creative risk. The agent must come up with a revolutionary new idea. If succesful (small probability) it yields huge payoffs. However, if completely unsuccesful it yields a very low payoff. If the agent does not take risks he is likely to achieve create an 'average' project with middle payoffs. However, there is a potential problem with such a contract: If wages decrease in output, this would encourage agents to hide or destroy output.