Limited Liability (lecture 3)

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  - But there are of course real constraints on liability (minimum wage, credit-constraints, social norms...)

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- Where the L-L constraint binds, the PC does not, and vice/versa. [Class question: Explain why.]

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  2. We can solve for this payment by meeting the IC exactly: $t_j^{SB} = \frac{\tilde{\psi}}{\pi_{j1}-\pi_{j0}}$,
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  3. If the likelihood ratios $\frac{\pi_{j1} - \pi_{j0}}{\pi_{j1}}$ are not decreasing in i (output), the payment is nondecreasing in production.
     Actually, if these likelihood ratios increase in output, the payment should be only to the highest output!

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We can look at it this way – incentives have a constant effect in their expected value – only because we assume a risk-neutral agent.