Limited Liability (lecture 3)

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Limited Liability?

- In previous slides, could the agent get a negative payment?
  - Yes: We only needed *expected* payments to be positive (above the reservation utility)
  - Remember: $\underline{t}^{SB} < t^* < \bar{t}^{SB}$, so if $t^* = r + \tilde{\psi}$ then $\underline{t}^{SB}$ could very well be negative.
- Is this realistic?
  - We really mean negative relative to the reservation wage, so it is conceivable
  - But there are of course real constraints on liability (minimum wage, credit-constraints, social norms...)

Note: with risk neutrality, for notational simplicity we can basically replace utility with income, and the utility cost of effort with an equivalent income cost, as utility must be an 'affine' transformation of income (explain). The only thing that needs adjusting is the effort cost; let $\tilde{\psi} = u^{-1}(\psi)$

Limited Liability and risk-neutrality

See treatment in L&M (4.3., 4.5.1) – Main points:

- Additional constraints: $t_i \geq -l$ (say $t_i \geq 0$) for all outputs i
- If binding, these L-L constraints requires that A gets a 'limited liability' rent if P wants to induce effort.
- Since he can't impose the 'stick' (punishment) he must offer the 'carrot' (reward) to have a sufficient difference between $\underline{t}^{SB}$ and $\bar{t}^{SB}$.
- Of course this additional cost of imposing effort means
  1. Effort binary: We require a higher social surplus from effort for A to want to induce effort
  2. Effort continuous: P will induce a lower level of effort than under the first-best. [Class question: Explain why, intuitively.]
- Where the L-L constraint binds, the PC does not, and vice/versa. [Class question: Explain why.]

- If we have many levels of output $(i = 1, ...n)$, A wants to induce effort, and LL binds (say, at 0) then:
  1. Bang-bang payments: P only gives a payment above 0 to the output that is most 'informative of high effort' (if unique), i.e., where $\frac{\pi_{j1}-\pi_{j0}}{\pi_{j1}}$ is highest
  2. We can solve for this payment by meeting the IC exactly:
     $t_j^{SB} = \frac{\tilde{\psi}}{\pi_{j1}-\pi_{j0}}$,
     this yields a limited liability rent: $EU^{SB} = \frac{\pi_{j1}\tilde{\psi}}{\pi_{j1}-\pi_{j0}}$
  3. If the likelihood ratios $\frac{\pi_{j1}-\pi_{j0}}{\pi_{j1}}$ are not decreasing in i (output), the payment is nondecreasing in production.
     Actually, if these likelihood ratios increase in output, the payment should be only to the highest output!

P doesn't like giving $t_i > -l$ in any contingency.

If he has to, he'd give this reward only to the output that is most informative of effort – most 'bang for the buck', and thus is most effective (per £) at inducing high effort.

I.e., he rewards the case that increases A's expected payment under high effort the most relative to it's increase in A's expected payment under low effort. This is represented by the numerator: $\pi_{j1} - \pi_{j0}$, while the expected cost to P (per £) increases linearly in the denominator π_{j1}.

So the expected incentive for A per £ spent by P is highest if the incentive goes only to the output where $\frac{\pi_{j1}-\pi_{j0}}{\pi_{j1}}$ is highest.

We can look at it this way – incentives have a constant effect in their expected value – only because we assume a risk-neutral agent.