Adverse Selection/ Hidden Information

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Reading(s): Laffont and Martimort ch. 2; “Proofs ” largely follow Salanie Ch. 2. No comparable treatment of this material in Milgrom and Roberts.

Practice problems up on CMR (I may add more)

Introduction: Adverse Selection (Hidden information) and screening

Optimal design of incentives when the agent has private information on a variable that affects the payoffs of the agent as well as the principal.

Motivational example:

'Pursuit of Happyness (link)'

Discuss:

- Why would Dean Witter make the internship unpaid? What kind of a person would take such an internship?
- Why would firm give such a huge return to its brokers?
- What are the inefficiencies associated with this?
- Who is bearing the risk?

Examples

1. Firm hires employee of unknown skill or unknown intrinsic motivation
2. Insurance company sells policy to individual of unknown health status
3. Govt. (or firm) procurement from (or regulation of) monopoly firm w/unknown mgnl. cost (e.g., NHS services)... Or within a firm (see theory of the firm)
4. Firm sells good to consumer w/ unknown value (2nd degree price discrim.)
5. Investor delegates management of his portfolio
6. A landlord delegates the cultivation of land with unknown productivity
7. Housemates decide which level of TV plan to buy (public good, mechanism design problem)

Key features: Principal would want to reach allocative efficiency, if he could extract all the surplus

- ... i.e., the proper matching of social marginal cost and marginal benefit ...
- ... but, where there is hidden information, this conflicts with the incentive compatibility constraints of the agent ...
- ... the principal may set a separate contract for each type, and hopes they will ‘self-select’ (this is called ‘screening’)...
- ...this leads to a second best set of contracts that reduces allocative efficiency to minimize the information rents paid to agents.

Basic Model (two types)

- Principal delegates production to agent: q units.
- U(Principal)= $S(q)$ is continuous and continuously differentiable (C^2?), i.e., with a continuous first derivative.
- Where $S(0) = 0$, $S'(0) > \bar{\theta} > 0$, and $S''(.) < 0$ (declining marginal valuation of production $\implies$ "risk aversion" over output).
- Fixed costs F (ignorable – the principal will cover this, assuming his expected utility exceeds this amount)
- Marginal cost $\theta \in \{\underline{\theta}, \bar{\theta}\}$, where $\underline{\theta} < \bar{\theta}$.

- U(agent)=$t - \theta q$ (her payment, net of costs)
- Total cost of production of q units is given by

$$
\begin{aligned}
C(q,\theta) &= F + \theta q \\
\theta &\in \{\bar{\theta}, \underline{\theta}\}
\end{aligned}
$$

Note: Production is not stochastic here $\Longrightarrow$ risk not an issue for A (but it will be for P, since type of agent unknown)

A Contract

1. P offers a TIOLI contract to A
2. Contract: a quantity q and a payment t corresponding to each quantity.
3. Set of all possible contracts: $A = \{(q, t) : q \in R^{+}, t \in R\}$

Principal's objective:

$$\max E(\Pi) = \Pi = \max_{q_{lc}, q_{hc}} E(S(q) - t(q))$$

$$s.t.\ t_{lc} - \underline{\theta} q_{lc} \geq 0$$

$$\text{and } t_{hc} - \bar{\theta} q_{hc} \geq 0$$

$$\text{for } \theta \in \{\underline{\theta}, \bar{\theta}\},\ q \in \{q_{lc}, q_{hc}\}$$

Offer a separate contract to each type – no need for self-selection because P can discriminate (assuming this is legally possible!).

...

Must induce (each type of) A to accept contract

No reason to overcompensate $A \Longrightarrow$ Constraint binds $\Longrightarrow t = \theta q \Longrightarrow$Maximize $S(q) - \theta q$

$\Longrightarrow q^{FB}(\theta)$ solves $S'(q^{FB}) = \theta$ for each type, (from strict concavity of S)

$\Longrightarrow$Optimal contracts $C^{FB} = \{q^{FB}(\theta), t = F + q^{FB}(\theta) \times \theta\}$; for $\theta \in \{\underline{\theta}, \bar{\theta}\}$.

In other words $t_{lc} = F + q_{lc}^{FB}\underline{\theta}$ and $q_{lc} = q_{lc}^{FB}$

and $t_{hc} = F + q_{hc}^{FB}\bar{\theta}$ and $q_{hc} = q_{hc}^{FB}$

Each guy is asked to produce at her efficient level and reimbursed her costs. This could be supported by a "'forcing" contract where, if she does not produce this amount, she gets nothing (or a very negative payoff).

Incomplete information Contracts

- *Principal's beliefs*: type $\underline{\theta}$ with probability v, type $\bar{\theta}$ with probability $1 - v$.
- P offers contracts $\mathbf{C}^{SB} = \{(q_{lc}, t_{lc}), (q_{hc}, t_{hc})\}$
- Each contract is implicitly supposed to attract a particular type of agent (self-selection).

Cannot achieve first-best

Theorem

$\mathbf{C}^{FB}$ *cannot be implemented here.*

Proof.

Sketch: Under $\mathbf{C}^{FB}$, both types of agent will strictly prefer to accept the contract $\{q_{hc}^{FB}, t_{hc} = F + q_{hc}^{FB}\bar{\theta}\}$. Not optimal for the principal if agent is of type $\underline{\theta}$.

Note: $q_{hc}^{FB}\bar{\theta} - q_{hc}^{FB}\underline{\theta} = (\bar{\theta} - \underline{\theta})q_{hc}^{FB} > 0$, so low-cost type's IC constraint not met at $\mathbf{C}^{FB}$. □

See L&M figure 2.2 through 2.5 for a graphical depiction.

IC and PC Constraints

A set of contracts $\{(q_{lc}, t_{lc}), (q_{hc}, t_{hc})\}$ is incentive compatible for type $\underline{\theta}$ if:

$$t_{lc} - \underline{\theta} q_{lc} \geq t_{hc} - \underline{\theta} q_{hc} \tag{IC($\underline{\theta}$)}$$

Similarly, a set of contracts $\{(q_{lc}, t_{lc}), (q_{hc}, t_{hc})$ is incentive compatible for type $\bar{\theta}$ if:

$$t_{hc} - \bar{\theta} q_{hc} \geq t_{lc} - \bar{\theta} q_{lc} \tag{IC($\bar{\theta}$)}$$

The corresponding participation constraints are simply that the contracts yield non-negative utility (reservation wage assumed to be 0):

$$t_{lc} - \underline{\theta} q_{lc} \geq 0 \tag{PC($\underline{\theta}$)}$$

$$t_{hc} - \bar{\theta} q_{hc} \geq 0 \tag{PC($\bar{\theta}$)}$$

Important Special Cases

1. Bunching/pooling: Single contract for both types. Incentive constraints trivially satisfied, and the PC of the $\underline{\theta}$ type is weaker than for the $\bar{\theta}$ type. So we only need to check one constraint for this type of contract.
2. Shut Down: A $\{0, 0\}$ contract for high type and a regular contract for the low cost type. This implicitly involves a shut down of the high cost type.

In general, we want to check that neither of these are superior to the interior, separating solutions derived below (but case 2 is checked automatically by the optimisation procedure, and case 1 is never optimal given our assumptions).

Theorem

Low cost type will produce at least as much .

Proof.

Adding up IC($\underline{\theta}$) and IC($\bar{\theta}$):

$$(t_{hc} + t_{lc}) - \bar{\theta} q_{hc} - \underline{\theta} q_{lc} \geq (t_{hc} + t_{lc}) - \bar{\theta} q_{lc} - \underline{\theta} q_{hc} \quad (1)$$

$$-\underline{\theta} q_{lc} + \bar{\theta} q_{lc} \geq -\underline{\theta} q_{hc} + \bar{\theta} q_{hc} \quad (2)$$

$$(\bar{\theta} - \underline{\theta}) q_{lc} \geq (\bar{\theta} - \underline{\theta}) q_{hc} \quad (3)$$

$$q_{lc} \geq q_{hc} \quad (4)$$

□

Information rents

Theorem

The low-cost type must receive a positive surplus – an "information rent' (in an interior, separating contract)

Proof

Sketch: The low-cost type can always mimic the high-cost type and earn a surplus ($\Delta\theta q_{hc}$ below) – we need to give her at least this in order to 'be herself."

Suppose $q_{hc}, q_{lc} > 0$

$\Longrightarrow u(\bar{\theta}, q_{hc}) = t_{hc} - \bar{\theta} q_{hc} \geq 0$ needed to satisfy the high-cost type's PC

$\Longrightarrow$If low type accepts this ('lies'), gets

$$\begin{aligned} u(\underline{\theta}, q_{hc}) &= t_{hc} - \underline{\theta} q_{hc} = t_{hc} - \bar{\theta} q_{hc} + \Delta\theta q_{hc} \\ &= u(\bar{\theta}, q_{hc}) + \Delta\theta q_{hc} > 0 \\ \text{Where } \Delta\theta &\equiv \bar{\theta} - \underline{\theta} \end{aligned}$$

Thus a positive surplus is needed to prevent 'mimicry,' i.e., $u(\underline{\theta}, q_{lc}) \geq u(\underline{\theta}, q_{hc}) > 0$

P's Optimization Problem (interior, separating)

$$\max_{q_{hc},t_{hc},q_{lc},t_{lc}} v[S(q_{lc}) - t_{lc}] + (1-v)[S(q_{hc}) - t_{hc}]$$
$$s.t.\ IC(\underline{\theta}),\ IC(\bar{\theta}),\ PC(\underline{\theta}), \text{ and } PC(\bar{\theta})$$

I.e. (writing this in terms of the net utilities of the agents) u_{lc} and u_{hc})

$$\max_{(q_{lc},t_{lc}),(q_{hc},t_{hc})} v[S(q_{lc}) - \underline{\theta}q_{lc}] + (1-v)[S(q_{hc}) - \bar{\theta}q_{hc}] - [vu_{lc} + (1-v)u_{hc}]$$

This breaks the objective function into the social value of trade minus the expected information rent.

Subject to

$$\begin{aligned} u_{lc} &\geq u_{hc} + \Delta\theta q_{hc} \\ u_{hc} &\geq u_{lc} - \Delta\theta q_{lc} \\ u_{lc} &\geq 0 \\ u_{hc} &\geq 0 \end{aligned}$$

Write on board:

$$
\begin{aligned}
PC(\underline{\theta}) &\quad : \quad t_{lc} - \underline{\theta} q_{lc} \equiv u_{lc} \geq 0 \\
PC(\bar{\theta}) &\quad : \quad t_{hc} - \bar{\theta} q_{hc} \equiv u_{hc} \geq 0 \\
IC\underline{(\theta)} &\quad : \quad t_{lc} - \underline{\theta} q_{lc} \geq t_{hc} - \underline{\theta} q_{hc}; \\
&\implies u_{lc} \geq u_{hc} + \Delta\theta q_{hc} \\
IC(\bar{\theta}) &\quad : \quad t_{hc} - \bar{\theta} q_{hc} \geq t_{lc} - \bar{\theta} q_{lc} \\
&\implies u_{hc} \geq u_{lc} - \Delta\theta q_{lc}
\end{aligned}
$$

Which constraints *do* bind?

...We can thus 'substitute them in', and simplify the problem.

Lemma

$PC(\bar{\theta})$ *is a binding constraint*

Proof.

(by contradiction) Suppose we have a contract meeting all the constraints, where $u_{hc} > 0$. Then P could lower u_{hc} by "ε", increase P's expected surplus, and the high-cost type would still participate.

Since $PC(\underline{\theta})$ doesn't bind (information rent) P could also lower u_{lc} by the same "ε", preserving $IC(\underline{\theta})$ and IC$(\bar{\theta})$. □

Lemma

$IC(\underline{\theta})$ is a binding constraint

Proof.

(by contradiction) If not, i.e., $u_{lc} > u_{hc} + \Delta\theta q_{hc}$, P could lower u_{lc}, doing better and still satisfying the relevant constraints:

$IC(\underline{\theta})$ would continue to hold if it didn't bind before.

$IC(\bar{\theta})$ is *relaxed* (*less* incentive for high-cost type to 'fake it').

$PC(\underline{\theta})$ would continue to hold because it didn't bind before– remember we showed an 'information rent.'

$PC(\bar{\theta})$ is unaffected. □

Thus:

We know $PC(\bar{\theta})$ and $IC(\underline{\theta})$ bind

$\Longrightarrow$Any optimal contract must have $u_{hc} = 0$,

i.e., $t_{hc} - \bar{\theta} q_{hc} = 0$

$\Longrightarrow t_{hc} = \bar{\theta} q_{hc}$

and $u_{lc} = u_{hc} + \Delta\theta q_{hc}$.

i.e., $t_{lc} - \underline{\theta} q_{lc} = 0 + \Delta\theta q_{hc}$

$\Longrightarrow t_{lc} = \underline{\theta} q_{lc} + \Delta\theta q_{hc}$

Which constraints *do not* bind?

$PC(\underline{\theta})$ does not bind (information rent) – so we can ignore it.

$IC(\bar{\theta})$ is redundant:

$$IC(\bar{\theta}) : u_{hc} \geq u_{lc} - \Delta\theta q_{lc}$$

$$u_{hc} = t_{hc} - \bar{\theta} q_{hc} = 0 \text{ since } PC(\bar{\theta}) \text{ binds.}$$

Recalling $u_{lc} = u_{hc} + \Delta\theta q_{hc}$ we have

$u_{lc} - \Delta\theta q_{lc} = u_{hc} + \Delta\theta(q_{hc} - q_{lc})$

$= 0 + \Delta\theta(q_{hc} - q_{lc}) \leq 0$

since $q_{hc} \leq q_{lc}$ (shown earlier)

$\Longrightarrow u_{hc} \geq u_{lc} - \Delta\theta q_{lc}$

So

$IC(\bar{\theta})$ holds as a result of the other constraints, it is redundant.

If $q_{hc} < q_{lc}$, which we will see later, then $u_{hc} > u_{lc} - \Delta\theta q_{lc}$, i.e., $IC(\bar{\theta})$ does not 'bind.'

Solving...

$\Longrightarrow$Optimization problem:

$$\begin{aligned} & \max_{q_{lc},q_{hc}} E(\Pi) \\ = & \max_{q_{lc},q_{hc}} E(S(q)-t(q)) \\ = & \max_{q_{lc},q_{hc}} v \times [S(q_{lc})-t_{lc}] + (1-v)[S(q_{hc})-t_{hc}] \\ & \text{s.t. PC \& IC constraints} \end{aligned}$$

Substituting in binding constraints – now 'unconstrained'

$$\max_{q_{lc},q_{hc}} v[S(q_{lc}) - \underline{\theta} q - \Delta\theta q_{hc}] + (1-v)[S(q_{hc}) - \bar{\theta} q_{hc}]$$

$$\max_{q_{lc},q_{hc}} \Pi = \max_{q_{lc},q_{hc}} v \times [S(q_{lc}) - \underline{\theta} q_{lc} - \Delta\theta q_{hc}] + (1-v)[S(q_{hc}) - \bar{\theta} q_{hc}]$$

Given that this is a concave problem, and assuming that the optimum is interior, the first order conditions are necessary and sufficient for the optimum:

$$\begin{aligned} \frac{\partial \Pi}{\partial q_{lc}} &= v \times [S'(q_{lc}) - \underline{\theta}] = 0 \\ &\Longrightarrow S'(q_{lc}^{SB}) = \underline{\theta} \\ &\Longrightarrow q_{lc}^{SB} = q_{lc}^{*} ... \Longrightarrow \text{'efficient!'} \end{aligned}$$

$$\begin{aligned} \frac{\partial \Pi}{\partial q_{hc}} &= (1-v)[S'(q_{hc}^{SB}) - \bar{\theta}] - v\Delta\theta = 0 \\ S'(q_{hc}^{SB}) &= \bar{\theta} + \frac{v}{(1-v)}\Delta\theta \; ... \Longrightarrow q_{hc}^{SB} < q_{hc}^{*} \end{aligned}$$

The optimal second-best contract thus offers the low cost type the first best quantity while it distorts the quantity of the high cost agent downward to minimize information rents.

This distortion increases in $\frac{v}{(1-v)}$ – the odds of a low-cost type – and in $\Delta\theta$ – the cost difference.

This also implies $q_{hc}^{SB} < q_{hc}^{*} < q_{lc}^{*} = q_{lc}^{SB}$ as claimed before, so $IC(\bar{\theta})$ holds with inequality.

But is it worth doing?

Profit:

$$E[\Pi^{SB}] = v[S(q_{lc}^*) - \underline{\theta} q_{lc}^* - \Delta\theta q_{hc}^*] + (1-v)[S(q_{hc}^{SB}) - \bar{\theta} q_{hc}^{SB}]$$

- There is a cost to inducing q_{hc}^{SB} and $q_{lc}^{SB} > q_{hc}^{SB}$
  - Fixed (we ignore here) and marginal costs of output
  - Information rents to low-cost type ($\Delta\theta q_{hc}^{SB}$)
  - (Other un-modeled costs, e.g., menu costs, psychological factors)

Alternative I – Bunching/pooling:

Single contract (1 output, one payment) for both types. Maximize s.t. $PC(\bar{\theta})$ only, which binds.

$$\Longrightarrow t^{pool} - \bar{\theta}q \equiv u_{hc}^{pool} = 0$$

$$\Longrightarrow t^{pool} = \bar{\theta}q$$

(Class question: why?)

solving pooling case

$$
\begin{aligned}
E[\Pi^{Pool}] &= S(q^{Pool}) - t^{pool} = S(q^{Pool}) - \bar{\theta} q^{pool} \\
q^{pool} &= \max_q S(q) - \bar{\theta} q \\
&\Longrightarrow S'(q^{Pool}) = \bar{\theta} \\
&\Longrightarrow q^{pool} = q^*_{hc} \\
&\Longrightarrow t^{pool} = \bar{\theta} q^*_{hc} \\
&\Longrightarrow E[\Pi^{pool}] = S(q^*_{hc}) - \bar{\theta} q^*_{hc} \\
u_{lc} &= \Delta \bar{\theta} q^*_{hc}
\end{aligned}
$$

The optimal pooling strategy has both agents producing at the level that is technically efficient for the high-cost type.

The high cost type will get no surplus, but the low-cost type gets a rent $\Delta\bar{\theta}q^*_{hc}$.

Note: this is a higher rent then in the separating case ($\Delta\bar{\theta}q^*_{hc}$ versus $\Delta\theta q^{SB}_{hc}$)!

Alternative II – 'Shut down' high-cost type (or both types)

Note: this resembles the classic "screening" contract; greater reward for higher effort means the average level of those who will accept the contract is higher! ... See 156-159 in Milgrom and Roberts

Maximize s.t. $PC(\underline{\theta})$ only, which binds. At this contract the high-cost type will not want to participate.

$$\Longrightarrow t^{shut} - \underline{\theta} q^{shut} \equiv u_{lc}^{shut} = 0$$

$$\Longrightarrow t^{shut} = \underline{\theta} q^{shut}$$

$$
\begin{aligned}
E[\Pi^{shut}] &= v \times \left(S(q^{shut}) - t^{shut}\right) = v \times \left(S(q^{shut}) - \underline{\theta} q^{shut}\right) \qquad (5) \\
q^{shut} &= \max_q v \times (S(q) - \underline{\theta} q) \\
&\implies S'(q^{shut}) = \underline{\theta} \\
&\implies q^{shut} = q^*_{lc} \\
&\implies t^{shut} = \underline{\theta} q^*_{lc} \\
&\implies E[\Pi^{shut}] = v \times (S(q^*_{lc}) - \underline{\theta} q^*_{lc})
\end{aligned}
$$

The optimal 'shut down' strategy has only the low-cost agent producing. Since there is no IC constraint it is obvious that P will get her to produce at her technically efficient level, and P will take all the rent.

Caveat: shut-down implied by a negative solution to the previous problem

Remember the FOC for the high-cost type's 'SB' output:

$$S'(q_{hc}^{SB}) = \bar{\theta} + [\frac{v}{(1-v)}\Delta\theta\ ] \tag{6}$$

The marginal benefit of the high-cost type's output was set equal to its technical cost plus a term for the effect of increasing this output on the rent that must be paid to the low-cost type.

But if the latter effect is large enough, this may have no positive solution. The first unit of the high-cost type's output may be too costly in net.

Thus $q_{hc}^{SB} = 0$ would be optimal, 'shutting down' the high-cost type.

$q_{hc}^{SB} = 0$ will also imply no rent for the low-cost type (remember, the rent was $\Delta\theta q_{hc}$).

Thus the 'shut down' strategy need not be checked separately in the continuous differentiable concave case (unless you are asked to do so).

Comparing alternatives – Interior-separating, pooling, shut-down one, shut down all

$$
\begin{aligned}
E[\Pi^{SB}] &= v[S(q_{lc}^*) - \underline{\theta} q_{lc}^* - \Delta\theta q_{hc}^*] + (1-v)[S(q_{hc}^{SB}) - \bar{\theta} q_{hc}^{SB}] \\
E[\Pi^{Pool}] &= S(q_{hc}^*) - \bar{\theta} q_{hc}^* \\
E[\Pi^{shut}] &= v\,(S(q_{lc}^*) - \underline{\theta} q_{lc}^*) \\
E[\Pi^{shut\ all}] &= 0
\end{aligned}
$$

In general, all of these possibilities should be compared.

In a very general case (although not given the assumptions here), any of the four could be the optimal one. [Note, the 'shut all' is not optimal given our assumptions above unless we add a fixed cost F to all the other alternative policies, or have positive reservation wages; Pooling also will not be optimal here, although it can if we allow different functional assumptions; see below.]

Note: These comparisons will be easier to make in a parametric or numerical case.

For example, for P to prefer to have both types of agent produce different positive output over only having the low-cost type produce, we require:

$$
\begin{aligned}
E[\Pi^{SB}] &> E[\Pi^{shut}] \Longleftrightarrow \\
v[S(q_{lc}^*) - \underline{\theta} q_{lc}^* - \Delta\theta q_{hc}^*] + (1-v)[S(q_{hc}^{SB}) - \bar{\theta} q_{hc}^{SB}] &> \\
v\left(S(q_{lc}^*) - \underline{\theta} q_{lc}^*\right) &\Longleftrightarrow \\
(1-v)[S(q_{hc}^{SB}) - \bar{\theta} q_{hc}^{SB}] &> v\Delta\theta q_{hc}^*
\end{aligned}
$$

I.e., the expected benefit of the high-cost type's output must exceed the resulting information rent paid to the low-cost type.

$$(1-v)[S(q_{hc}^{SB})-\bar{\theta}q_{hc}^{SB}] > v\Delta\theta q_{hc}^{*}$$

The relative advantage of writing a contract that only the low-cost type will sign increases as:

1. The difference in costs increases.
2. Low-cost types become more likely.
3. The value of additional output decreases.

Comparing the pooling and shutting-down possibilities:

$$
\begin{aligned}
E[\Pi^{shut}] &> E[\Pi^{Pool}] \\
\iff & v\left(S(q_{lc}^*) - \underline{\theta} q_{lc}^*\right) > \left(S(q_{hc}^*) - \bar{\theta} q_{hc}^*\right)
\end{aligned}
$$

The 'net value' generated by a low-cost type (weighted by the probability that a low-cost type is drawn) must exceed the 'net value' generated by a high cost type.

Comparing pooling and interior-separating (SB) contracts:

$$\begin{aligned} E[\Pi^{SB}] &> E[\Pi^{pool}] \\ v[S(q^*_{lc}) - \underline{\theta} q^*_{lc} - \Delta\theta q^*_{hc}] + (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}] &> S(q^*_{hc}) - \bar{\theta} q^*_{hc} \end{aligned}$$

In the 'SB' contract P pays a rent to the low-cost type (a larger rent than in the pooling case), and gets less production from the high-cost type ('distortion'). However, P gets a higher output from the low-cost type under the 'SB' contract than under the pooling contract.

It appears unclear which contract P prefers. It will depend on the probability of a low-cost type, the relative costs of the two types, and the value of additional output (the $S(\cdot)$ function). BUT with the conditions as given, we know pooling on a positive output will never be optimal! [see next slide]

Proof of no pooling

Proof by contradiction

We compared $v[S(q^*_{lc}) - \underline{\theta} q^*_{lc} - \Delta\theta q^*_{hc}] + (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}]$ and $S(q^*_{hc}) - \bar{\theta} q^*_{hc}$, a messy comparison. But, we need only show that the pooling contract can never be optimal, i.e.., show that if P is 'pooling on positive output,' then he could do better.

...no pooling proof continued

Suppose the "best possible" pooling, yielding $E[\Pi^{pool}] = S(q^*_{hc}) - \bar{\theta} q^*_{hc}$. But to this single contract $\{q^*_{hc}, \bar{\theta} q^*_{hc}\}$, P could introduce a second contract $q_{lc} = q^*_{hc} + \varepsilon$ and reward anyone who takes it an additional $\underline{\theta}\varepsilon$. By convention, the low-cost type will take this contract (because she is indifferent and P wants her to, as we will see), Obviously the high-cost type will not. Thus, P is better off doing so (note, payoffs will only vary when a low-cost type arises) as long as $s(q^*_{hc} + \varepsilon) - s(q^*_{hc}) > \underline{\theta}\varepsilon$ for some small ε, i.e., as long as $s'(q^*_{hc}) > \underline{\theta}$ (as long as s' is continuous, as assumed).

By the definition $s'(q^*_{hc}) = \bar{\theta} > \underline{\theta}$. Hence, $s'(q^*_{hc}) > \underline{\theta}$, and the principal can improve on a pooling contract.

But:

However, pooling may be optimal under other functional assumptions, e.g., if principal's utility is $x - t$ and the agent's utility is $t - \theta x^{\frac{1}{2}}$.

Overall results

Theorem

Second best transfers are given by

$$t_{lc} = \underline{\theta} q_{lc} + \Delta\theta q_{hc}; \quad t_{hc} = \bar{\theta} q_{hc}. \tag{7}$$

The efficient type gets a positive information rent, given by $u_{lc} = \Delta\theta q_{hc}$ *. By making the high-cost agent produce less, and paying her less, we make her target output less attractive to the low-cost agent. Output distortion of inefficient type:*

$$S'(q_{hc}) = \bar{\theta} + \frac{v}{1-v}\Delta\theta. \tag{8}$$

No output distortion of the low cost type $q_{lc} = q_{lc}^{FB}$

Thus under asymmetric information, the optimal menu of contracts entails:

- No output distortion of the low cost type, thus $q_{lc} = q_{lc}^{FB}$
- Downward output distortion of inefficient type:

$$S'(q_{hc}) = \bar{\theta} + \frac{v}{1-v}\Delta\theta. \tag{9}$$

'by making the high-cost agent produce less, and paying him less, we make his target output less attractive to the low cost agent'

...

- Only efficient type gets a positive information rent, given by $u_{lc} = \Delta\theta q_{hc}$.
- Second best transfers are given by

$$t_{lc} = \underline{\theta} q_{lc} + \Delta\theta q_{hc}; \quad t_{hc} = \bar{\theta} q_{hc}. \tag{10}$$

Remember:

- Which agent's PC constraint binds, and why?
- Which agent's IC constraint binds, and why?
- When does the principal want to meet these constraints (interior, separating equilibrium)?