Adverse Selection/ Hidden Information

David Reinstein

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Reading(s): Laffont and Martimort ch. 2; "Proofs " largely follow Salanie Ch. 2. No comparable treatment of this material in Milgrom and Roberts.

Practice problems up on CMR (I may add more)

Introduction: Adverse Selection (Hidden information) and screening

Optimal design of incentives when the agent has private information on a variable that affects the payoffs of the agent as well as the principal.

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- Who is bearing the risk?

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6. A landlord delegates the cultivation of land with unknown productivity
7. Housemates decide which level of TV plan to buy (public good, mechanism design problem)

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- ... the principal may set a separate contract for each type, and hopes they will 'self-select' (this is called 'screening')...
- ...this leads to a second best set of contracts that reduces allocative efficiency to minimize the information rents paid to agents.

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- Marginal cost $\theta \in \{\underline{\theta}, \bar{\theta}\}$, where $\underline{\theta} < \bar{\theta}$.

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Note: Production is not stochastic here $\Longrightarrow$ risk not an issue for A (but it will be for P, since type of agent unknown)

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3. Set of all possible contracts: $A = \{(q,t) : q \in R^+, t \in R\}$

Principal's objective:

$$\max E(\Pi) = \Pi = \max_{q_{lc}, q_{hc}} E(S(q) - t(q))$$

$$s.t.\ t_{lc} - \underline{\theta} q_{lc} \geq 0$$

$$\text{and } t_{hc} - \bar{\theta} q_{hc} \geq 0$$

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Offer a separate contract to each type – no need for self-selection because P can discriminate (assuming this is legally possible!).

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$\Longrightarrow q^{FB}(\theta)$ solves $S'(q^{FB}) = \theta$ for each type, (from strict concavity of S)

$\Longrightarrow$Optimal contracts $C^{FB} = \{q^{FB}(\theta), t = F + q^{FB}(\theta) \times \theta\}$; for $\theta \in \{\underline{\theta}, \bar{\theta}\}$.

In other words $t_{lc} = F + q_{lc}^{FB}\underline{\theta}$ and $q_{lc} = q_{lc}^{FB}$

and $t_{hc} = F + q_{hc}^{FB}\bar{\theta}$ and $q_{hc} = q_{hc}^{FB}$

Each guy is asked to produce at her efficient level and reimbursed her costs. This could be supported by a "'forcing" contract where, if she does not produce this amount, she gets nothing (or a very negative payoff).

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- Each contract is implicitly supposed to attract a particular type of agent (self-selection).

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Proof.

Sketch: Under $\mathbf{C}^{FB}$, both types of agent will strictly prefer to accept the contract $\{q_{hc}^{FB}, t_{hc} = F + q_{hc}^{FB}\bar{\theta}\}$. Not optimal for the principal if agent is of type $\underline{\theta}$.

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Note: $q_{hc}^{FB}\bar{\theta} - q_{hc}^{FB}\underline{\theta} = (\bar{\theta} - \underline{\theta})q_{hc}^{FB} > 0$, so low-cost type's IC constraint not met at $\mathbf{C}^{FB}$. □

See L&M figure 2.2 through 2.5 for a graphical depiction.

IC and PC Constraints

A set of contracts $\{(q_{lc}, t_{lc}), (q_{hc}, t_{hc})\}$ is incentive compatible for type $\underline{\theta}$ if:

$$t_{lc} - \underline{\theta} q_{lc} \geq t_{hc} - \underline{\theta} q_{hc} \qquad \text{(IC}(\underline{\theta}))$$

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The corresponding participation constraints are simply that the contracts yield non-negative utility (reservation wage assumed to be 0):

$$t_{lc} - \underline{\theta} q_{lc} \geq 0 \tag{PC($\underline{\theta}$)}$$

$$t_{hc} - \bar{\theta} q_{hc} \geq 0 \tag{PC($\bar{\theta}$)}$$

Important Special Cases

1. Bunching/pooling: Single contract for both types. Incentive constraints trivially satisfied, and the PC of the $\underline{\theta}$ type is weaker than for the $\bar{\theta}$ type. So we only need to check one constraint for this type of contract.

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In general, we want to check that neither of these are superior to the interior, separating solutions derived below (but case 2 is checked automatically by the optimisation procedure, and case 1 is never optimal given our assumptions).

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$$(t_{hc} + t_{lc}) - \bar{\theta} q_{hc} - \underline{\theta} q_{lc} \quad \geq \quad (t_{hc} + t_{lc}) - \bar{\theta} q_{lc} - \underline{\theta} q_{hc}$$

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$$-\underline{\theta} q_{lc} + \bar{\theta} q_{lc} \geq -\underline{\theta} q_{hc} + \bar{\theta} q_{hc} \quad (2)$$

$$(\bar{\theta} - \underline{\theta}) q_{lc} \geq (\bar{\theta} - \underline{\theta}) q_{hc} \quad (3)$$

$$q_{lc} \geq q_{hc} \quad (4)$$

□

Information rents

Theorem

The low-cost type must receive a positive surplus – an "information rent' (in an interior, separating contract)

Proof

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$$\begin{aligned} u(\underline{\theta}, q_{hc}) &= t_{hc} - \underline{\theta} q_{hc} = t_{hc} - \bar{\theta} q_{hc} + \Delta\theta q_{hc} \\ &= u(\bar{\theta}, q_{hc}) + \Delta\theta q_{hc} > 0 \\ \text{Where } \Delta\theta &\equiv \bar{\theta} - \underline{\theta} \end{aligned}$$

Thus a positive surplus is needed to prevent 'mimicry,' i.e., $u(\underline{\theta}, q_{lc}) \geq u(\underline{\theta}, q_{hc}) > 0$

P's Optimization Problem (interior, separating)

$$\max_{q_{hc},t_{hc},q_{lc},t_{lc}} v[S(q_{lc}) - t_{lc}] + (1-v)[S(q_{hc}) - t_{hc}]$$

$s.t.$ $IC(\underline{\theta})$, $IC(\bar{\theta})$, $PC(\underline{\theta})$, and $PC(\bar{\theta})$

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Subject to

$$\begin{aligned} u_{lc} &\geq u_{hc} + \Delta\theta q_{hc} \\ u_{hc} &\geq u_{lc} - \Delta\theta q_{lc} \\ u_{lc} &\geq 0 \\ u_{hc} &\geq 0 \end{aligned}$$

Write on board:

$$\begin{aligned}
PC(\underline{\theta}) &: \quad t_{lc} - \underline{\theta} q_{lc} \equiv u_{lc} \geq 0 \\
PC(\bar{\theta}) &: \quad t_{hc} - \bar{\theta} q_{hc} \equiv u_{hc} \geq 0 \\
IC\underline{(\theta)} &: \quad t_{lc} - \underline{\theta} q_{lc} \geq t_{hc} - \underline{\theta} q_{hc}; \\
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Since $PC(\underline{\theta})$ doesn't bind (information rent) P could also lower u_{lc} by the same "ε", preserving $IC(\underline{\theta})$ and IC($\bar{\theta}$). □

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$PC(\bar{\theta})$ is unaffected. □

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and $u_{lc} = u_{hc} + \Delta\theta q_{hc}$.

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So

$IC(\bar{\theta})$ holds as a result of the other constraints, it is redundant.

If $q_{hc} < q_{lc}$, which we will see later, then $u_{hc} > u_{lc} - \Delta\theta q_{lc}$, i.e., $IC(\bar{\theta})$ does not 'bind.'

Solving...

$\Longrightarrow$Optimization problem:

$$\begin{aligned}
& \max_{q_{lc},q_{hc}} E(\Pi) \\
= & \max_{q_{lc},q_{hc}} E(S(q) - t(q)) \\
= & \max_{q_{lc},q_{hc}} v \times [S(q_{lc}) - t_{lc}] + (1 - v)[S(q_{hc}) - t_{hc}] \\
& \text{s.t. PC \& IC constraints}
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Substituting in binding constraints – now 'unconstrained'

$$\max_{q_{lc},q_{hc}} v[S(q_{lc})-\underline{\theta}q-\Delta\theta q_{hc}]+(1-v)[S(q_{hc})-\bar{\theta}q_{hc}]$$

$$\max_{q_{lc},q_{hc}} \Pi = \max_{q_{lc},q_{hc}} \; v \times [S(q_{lc}) - \underline{\theta} q_{lc} - \Delta\theta q_{hc}] + (1-v)[S(q_{hc}) - \bar{\theta} q_{hc}]$$

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S'(q_{hc}^{SB}) &= \bar{\theta} + \frac{v}{(1-v)}\Delta\theta \; ... \Longrightarrow q_{hc}^{SB} < q_{hc}^{*}
\end{aligned}
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The optimal second-best contract thus offers the low cost type the first best quantity while it distorts the quantity of the high cost agent downward to minimize information rents.

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This distortion increases in $\frac{v}{(1-v)}$ – the odds of a low-cost type – and in $\Delta\theta$ – the cost difference.

This also implies $q_{hc}^{SB} < q_{hc}^{*} < q_{lc}^{*} = q_{lc}^{SB}$ as claimed before, so $IC(\bar{\theta})$ holds with inequality.

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Profit:

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- There is a cost to inducing q^{SB}_{hc} and $q^{SB}_{lc} > q^{SB}_{hc}$
  - Fixed (we ignore here) and marginal costs of output
  - Information rents to low-cost type ($\Delta\theta q^{SB}_{hc}$)
  - (Other un-modeled costs, e.g., menu costs, psychological factors)

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Single contract (1 output, one payment) for both types. Maximize s.t. $PC(\bar{\theta})$ only, which binds.

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(Class question: why?)

solving pooling case

$$\begin{aligned}
E[\Pi^{Pool}] &= S(q^{Pool}) - t^{pool} = S(q^{Pool}) - \bar{\theta} q^{pool} \\
q^{pool} &= \max_q S(q) - \bar{\theta} q \\
&\implies S'(q^{Pool}) = \bar{\theta} \\
&\implies q^{pool} = q^*_{hc} \\
&\implies t^{pool} = \bar{\theta} q^*_{hc} \\
&\implies E[\Pi^{pool}] = S(q^*_{hc}) - \bar{\theta} q^*_{hc} \\
u_{lc} &= \Delta \bar{\theta} q^*_{hc}
\end{aligned}$$

The optimal pooling strategy has both agents producing at the level that is technically efficient for the high-cost type.

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Note: this is a higher rent then in the separating case ($\Delta\bar{\theta}q_{hc}^{*}$ versus $\Delta\theta q_{hc}^{SB}$)!

Alternative II – 'Shut down' high-cost type (or both types)

Note: this resembles the classic "screening" contract; greater reward for higher effort means the average level of those who will accept the contract is higher! ... See 156-159 in Milgrom and Roberts

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The optimal 'shut down' strategy has only the low-cost agent producing. Since there is no IC constraint it is obvious that P will get her to produce at her technically efficient level, and P will take all the rent.

Caveat: shut-down implied by a negative solution to the previous problem

Remember the FOC for the high-cost type's 'SB' output:

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$$S'(q_{hc}^{SB}) = \bar{\theta} + [\frac{v}{(1-v)}\Delta\theta\ ] \tag{6}$$

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Remember the FOC for the high-cost type's 'SB' output:

$$S'(q_{hc}^{SB}) = \bar{\theta} + [\frac{v}{(1-v)}\Delta\theta\ ] \tag{6}$$

The marginal benefit of the high-cost type's output was set equal to its technical cost plus a term for the effect of increasing this output on the rent that must be paid to the low-cost type.

But if the latter effect is large enough, this may have no positive solution. The first unit of the high-cost type's output may be too costly in net.

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Thus $q_{hc}^{SB} = 0$ would be optimal, 'shutting down' the high-cost type.

$q_{hc}^{SB} = 0$ will also imply no rent for the low-cost type (remember, the rent was $\Delta\theta q_{hc}$).

Thus the 'shut down' strategy need not be checked separately in the continuous differentiable concave case (unless you are asked to do so).

Comparing alternatives – Interior-separating, pooling, shut-down one, shut down all

$$
\begin{aligned}
E[\Pi^{SB}] &= v[S(q_{lc}^*) - \underline{\theta} q_{lc}^* - \Delta\theta q_{hc}^*] + (1-v)[S(q_{hc}^{SB}) - \bar{\theta} q_{hc}^{SB}] \\
E[\Pi^{Pool}] &= S(q_{hc}^*) - \bar{\theta} q_{hc}^* \\
E[\Pi^{shut}] &= v\,(S(q_{lc}^*) - \underline{\theta} q_{lc}^*) \\
E[\Pi^{shut\ all}] &= 0
\end{aligned}
$$

In general, all of these possibilities should be compared.

In a very general case (although not given the assumptions here), any of the four could be the optimal one. [Note, the ‘shut all’ is not optimal given our assumptions above unless we add a fixed cost F to all the other alternative policies, or have positive reservation wages; Pooling also will not be optimal here, although it can if we allow different functional assumptions; see below.]

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In a very general case (although not given the assumptions here), any of the four could be the optimal one. [Note, the 'shut all' is not optimal given our assumptions above unless we add a fixed cost F to all the other alternative policies, or have positive reservation wages; Pooling also will not be optimal here, although it can if we allow different functional assumptions; see below.]

Note: These comparisons will be easier to make in a parametric or numerical case.

For example, for P to prefer to have both types of agent produce different positive output over only having the low-cost type produce, we require:

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$$
\begin{aligned}
E[\Pi^{SB}] &> E[\Pi^{shut}] \iff \\
v[S(q^*_{lc}) - \underline{\theta} q^*_{lc} - \Delta\theta q^*_{hc}] + (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}] &> \\
v\left(S(q^*_{lc}) - \underline{\theta} q^*_{lc}\right) &\iff
\end{aligned}
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For example, for P to prefer to have both types of agent produce different positive output over only having the low-cost type produce, we require:

$$\begin{aligned} E[\Pi^{SB}] &> E[\Pi^{shut}] \iff \\ v[S(q^*_{lc}) - \underline{\theta} q^*_{lc} - \Delta\theta q^*_{hc}] + (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}] &> \\ v\left(S(q^*_{lc}) - \underline{\theta} q^*_{lc}\right) &\iff \\ (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}] &> v\Delta\theta q^*_{hc} \end{aligned}$$

I.e., the expected benefit of the high-cost type's output must exceed the resulting information rent paid to the low-cost type.

$$(1-v)[S(q_{hc}^{SB})-\bar{\theta}q_{hc}^{SB}] > v\Delta\theta q_{hc}^{*}$$

The relative advantage of writing a contract that only the low-cost type will sign increases as:

1. The difference in costs increases.

$$(1-v)[S(q_{hc}^{SB}) - \bar{\theta}q_{hc}^{SB}] > v\Delta\theta q_{hc}^{*}$$

The relative advantage of writing a contract that only the low-cost type will sign increases as:

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The relative advantage of writing a contract that only the low-cost type will sign increases as:

1. The difference in costs increases.
2. Low-cost types become more likely.
3. The value of additional output decreases.

Comparing the pooling and shutting-down possibilities:

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\end{aligned}
$$

The ‘net value’ generated by a low-cost type (weighted by the probability that a low-cost type is drawn) must exceed the ‘net value’ generated by a high cost type.

Comparing pooling and interior-separating (SB) contracts:

$$\begin{aligned} E[\Pi^{SB}] &> E[\Pi^{pool}] \\ v[S(q^*_{lc}) - \underline{\theta} q^*_{lc} - \Delta\theta q^*_{hc}] + (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}] &> S(q^*_{hc}) - \bar{\theta} q^*_{hc} \end{aligned}$$

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\end{aligned}
$$

In the 'SB' contract P pays a rent to the low-cost type (a larger rent than in the pooling case), and gets less production from the high-cost type ('distortion'). However, P gets a higher output from the low-cost type under the 'SB' contract than under the pooling contract.

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In the 'SB' contract P pays a rent to the low-cost type (a larger rent than in the pooling case), and gets less production from the high-cost type ('distortion'). However, P gets a higher output from the low-cost type under the 'SB' contract than under the pooling contract.

It appears unclear which contract P prefers. It will depend on the probability of a low-cost type, the relative costs of the two types, and the value of additional output (the $S(\cdot)$ function). BUT with the conditions as given, we know pooling on a positive output will never be optimal! [see next slide]

Proof of no pooling

Proof by contradiction

We compared $v[S(q^*_{lc}) - \underline{\theta} q^*_{lc} - \Delta\theta q^*_{hc}] + (1-v)[S(q^{SB}_{hc}) - \bar{\theta} q^{SB}_{hc}]$ and $S(q^*_{hc}) - \bar{\theta} q^*_{hc}$, a messy comparison. But, we need only show that the pooling contract can never be optimal, i.e.., show that if P is 'pooling on positive output,' then he could do better.

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By the definition $s'(q^*_{hc}) = \bar{\theta} > \underline{\theta}$. Hence, $s'(q^*_{hc}) > \underline{\theta}$, and the principal can improve on a pooling contract.

But:

However, pooling may be optimal under other functional assumptions, e.g., if principal's utility is $x - t$ and the agent's utility is $t - \theta x^{\frac{1}{2}}$.

Overall results

Theorem

Second best transfers are given by

$$t_{lc} = \underline{\theta} q_{lc} + \Delta\theta q_{hc}; \quad t_{hc} = \bar{\theta} q_{hc}. \tag{7}$$

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$$t_{lc} = \underline{\theta} q_{lc} + \Delta\theta q_{hc}; \quad t_{hc} = \bar{\theta} q_{hc}. \tag{10}$$

Remember:

- Which agent's PC constraint binds, and why?
- Which agent's IC constraint binds, and why?
- When does the principal want to meet these constraints (interior, separating equilibrium)?