Lecture 9: The Theories of the Firm

Largely from Gibbons (2004) and Goyal's lecture notes

David Reinstein

EC951

December 2011

Introduction: Coase's Bold Question

- Many transactions within 'markets'

Introduction: Coase's Bold Question

- Many transactions within 'markets'
- Many within firms, not market

Introduction: Coase's Bold Question

- Many transactions within 'markets'
- Many within firms, not market
- What are the factors that determine the boundaries of firms?

Introduction: Coase's Bold Question

- Many transactions within 'markets'
- Many within firms, not market
- What are the factors that determine the boundaries of firms?
- Managers, governments: The 'make or buy' problem

Introduction: Coase's Bold Question

- Many transactions within 'markets'
- Many within firms, not market
- What are the factors that determine the boundaries of firms?
- Managers, governments: The 'make or buy' problem
- Regulators: When is vertical or horizontal integration socially efficient?

What is a "complete" theory of the firm?

- Must explain why "all types" of firms exist in "all" relevant situations
- Must explain the *scope* of a firm
  - Ideally, there must be some *disadvantage* of size that, for a large enough firm, outweighs any advantage! I will refer to this as "*crossing over*"

Some key issues: Classical

- Economies of scale and scope

Some key issues: Classical

- Economies of scale and scope
- Double marginalization

Some key issues: Classical

- Economies of scale and scope
- Double marginalization
- Market power (over consumers and other firms)

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing

Some key issues: Modern;
Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "
- Ownership as right of residual control

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "
- Ownership as right of residual control
- Relationship-specific investments and the hold-up problem

Some key issues: Modern;
Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "
- Ownership as right of residual control
- Relationship-specific investments and the hold-up problem
- Rent-seeking investments

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "
- Ownership as right of residual control
- Relationship-specific investments and the hold-up problem
- Rent-seeking investments
- A noncontractible asset as an 'instrument' in a multi-task incentive problem

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "
- Ownership as right of residual control
- Relationship-specific investments and the hold-up problem
- Rent-seeking investments
- A noncontractible asset as an 'instrument' in a multi-task incentive problem
- 'Adaptive, sequential decision-making'

Some key issues: Modern;

Looking inside the firm...

- Incentives of management and transfer pricing
- Incomplete contracting
- "Socially destructive haggling over 'appropriable quasi-rents' "
- Ownership as right of residual control
- Relationship-specific investments and the hold-up problem
- Rent-seeking investments
- A noncontractible asset as an 'instrument' in a multi-task incentive problem
- 'Adaptive, sequential decision-making'
- Relational adaptation

Readings

1. R. Coase (1937), “The nature of the firm,” Economica.

2. **O. Hart and S. Grossman** (1986), The costs and benefits of integration: A Theory of Vertical and Lateral Integration, *The Journal of Political Economy*.

3. **R. Gibbons** (2004), *Four Formalizable theories of the firm*. mimeo, MIT.

4. O. Hart (1995), Firms, Contracts and Financial Structure. Oxford University Press. Chapters 1-4.

5. Tirole, Jean, The Theory of Industrial Organization, Chapter 1.

Readings

1. R. Coase (1937), "The nature of the firm," Economica.

2. **O. Hart and S. Grossman** (1986), The costs and benefits of integration: A Theory of Vertical and Lateral Integration, *The Journal of Political Economy*.

3. **R. Gibbons** (2004), *Four Formalizable theories of the firm*. mimeo, MIT.

4. O. Hart (1995), Firms, Contracts and Financial Structure. Oxford University Press. Chapters 1-4.

5. Tirole, Jean, The Theory of Industrial Organization, Chapter 1.

For exam: know competing theories of the firm, know Hart and Grossman (incomplete contracting) type model well.

What is a firm?

- The unit of production?

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
  - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
  - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.
  - We don't think of these all as one firm

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
    - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.
    - We don't think of these all as one firm
- A legal entity.

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
  - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.
  - We don't think of these all as one firm
- A legal entity.
  - But "firms" came before and come without legal registration.

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
    - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.
    - We don't think of these all as one firm
- A legal entity.
    - But "firms" came before and come without legal registration.
- A collection of 'employees'?

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
  - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.
  - We don't think of these all as one firm
- A legal entity.
  - But "firms" came before and come without legal registration.
- A collection of 'employees'?
  - Not sufficiently descriptive

The relevant explanation for firms should depend on the definition chosen.

What is a firm?

- The unit of production?
  - But several groups collaborate to produce the same products – e.g., a car can be made with parts from hundreds of different companies, and sold at dealer run by a private party.
  - We don't think of these all as one firm
- A legal entity.
  - But "firms" came before and come without legal registration.
- A collection of 'employees'?
  - Not sufficiently descriptive
- Grossman and Hart: 'We define a firm to consist of those assets that it owns or over which it has control.'

The relevant explanation for firms should depend on the definition chosen.

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; 'subadditivity'

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
    - With multiple goods there are more technical definitions; 'subadditivity'
- If all factors are replicable, avg. costs should decline monotonically in output

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; 'subadditivity'
- If all factors are replicable, avg. costs should decline monotonically in output
- If some factors are fixed (e.g., managerial talent), average costs will eventually rise.

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; ‘subadditivity’
- If all factors are replicable, avg. costs should decline monotonically in output
- If some factors are fixed (e.g., managerial talent), average costs will eventually rise.
  - But why can’t they hire a second manager?!

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; ‘subadditivity’
- If all factors are replicable, avg. costs should decline monotonically in output
- If some factors are fixed (e.g., managerial talent), average costs will eventually rise.
  - But why can’t they hire a second manager?!
  - Theory predicts each firm producing Q^* (at minimum average cost) in equilibrium

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; 'subadditivity'
- If all factors are replicable, avg. costs should decline monotonically in output
- If some factors are fixed (e.g., managerial talent), average costs will eventually rise.
  - But why can't they hire a second manager?!
  - Theory predicts each firm producing Q^* (at minimum average cost) in equilibrium
  - But why not have a single large firm with two divisions each producing Q^*?, i.e., why do the additional managers need to be employed *outside* the firm?

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; 'subadditivity'
- If all factors are replicable, avg. costs should decline monotonically in output
- If some factors are fixed (e.g., managerial talent), average costs will eventually rise.
  - But why can't they hire a second manager?!
  - Theory predicts each firm producing Q^* (at minimum average cost) in equilibrium
  - But why not have a single large firm with two divisions each producing Q^*?, i.e., why do the additional managers need to be employed *outside* the firm?
- Hart: 'Neoclassical theory is consistent with there being one huge firm in the world ... [or] with every plant and division of an existing firm becoming a separate and independent firm.'

Neoclassical & Industrial Organization Theory:

Economies of scale and scope

- With fixed costs, average costs should decline in output.
  - With multiple goods there are more technical definitions; 'subadditivity'
- If all factors are replicable, avg. costs should decline monotonically in output
- If some factors are fixed (e.g., managerial talent), average costs will eventually rise.
  - But why can't they hire a second manager?!
  - Theory predicts each firm producing Q^* (at minimum average cost) in equilibrium
  - But why not have a single large firm with two divisions each producing Q^*?, i.e., why do the additional managers need to be employed *outside* the firm?
- Hart: 'Neoclassical theory is consistent with there being one huge firm in the world ... [or] with every plant and division of an existing firm becoming a separate and independent firm.'
- On the other hand, why do we need a "firm" to achieve this scale: different entities can cooperate and band together without coming under one owner.

Market Power and integration

Double marginalization – example with one upstream and one downstream firm; both monopolies

Upstream firm produces a single input at zero cost, which is converted one-to-one into final output by the downstream firm.

Upstream firm sets a price for these inputs and the downstream firm chooses how much inputs to buy.

Demand is based on final market demand and the price of the input.

Market Power and integration

Double marginalization – example with one upstream and one downstream firm; both monopolies

Upstream firm produces a single input at zero cost, which is converted one-to-one into final output by the downstream firm.
Upstream firm sets a price for these inputs and the downstream firm chooses how much inputs to buy.
Demand is based on final market demand and the price of the input.

Assume that the downstream faces a linear demand: $Q = 1 - P$.

Thus, upstream firm will set price: $p_u = 1/2$, downstream firm will demand $q = 1/4$, total profits of the firms are given by $3/16$.

But the firms do better by integrating: if a single firm sets the optimal input price of 0 and final quantity of $1/2$ then profit of $1/4$ can be attained.

Moral: Markups over marginal cost cause inefficient behavior even for producers.

But even if the firms integrate this problem may persist, depending on how they structure transfer prices! So a "firm" is not sufficient.
–This is analogous to the government's problem in regulating a natural monopoly.

Moral: Markups over marginal cost cause inefficient behavior even for producers.

But even if the firms integrate this problem may persist, depending on how they structure transfer prices! So a "firm" is not sufficient.
–This is analogous to the government's problem in regulating a natural monopoly.

Also, this could alternately be solved with a discriminatory price (two-part tariff). So a "firm" may not be necessary.

Modeling double marginalization

Vertically integrated quantity and price set to maximize:

$$(p - c)D(p)$$

Modeling double marginalization

Vertically integrated quantity and price set to maximize:

$$(p-c)D(p)$$
$$\text{...(FOC...} \implies \text{MR=MC)}$$

Modeling double marginalization

Vertically integrated quantity and price set to maximize:

$$
\begin{aligned}
& (p-c)D(p) \\
\text{...(FOC...} \quad \Longrightarrow \quad & \text{MR=MC)} \\
\Longrightarrow \quad & p_I^* = -\frac{D(p)}{D'(p)} + c
\end{aligned}
$$

Modeling double marginalization

Vertically integrated quantity and price set to maximize:

$$(p - c)D(p)$$

$$\text{...(FOC...} \implies \text{MR=MC)}$$

$$\implies p_I^* = -\frac{D(p)}{D'(p)} + c$$

If the production requires buying an input from an upstream firm whose cost to produce this is c, the retailer (downstream firm) will set price to maximize:

$$(p_R - p^u(c))D(p_R)$$

$$\text{...(FOC...} \implies \text{MR=MC)}$$

Modeling double marginalization

Vertically integrated quantity and price set to maximize:

$$
\begin{aligned}
& (p-c)D(p) \\
\text{...(FOC...} \implies & \text{ MR=MC)} \\
\implies & p_I^* = -\frac{D(p)}{D'(p)} + c
\end{aligned}
$$

If the production requires buying an input from an upstream firm whose cost to produce this is c, the retailer (downstream firm) will set price to maximize:

$$
\begin{aligned}
& (p_R - p^u(c))D(p_R) \\
\text{...(FOC...} \implies & \text{ MR=MC)} \\
\implies & p_R^* = -\frac{D(p_R)}{D'(p_R)} + p^u(c)
\end{aligned}
$$

Where p^u is the upstream price of this input. Note that if the upstream firm has market power then $p^u(c) > c$ – it will charge a markup.

If the upstream firm is a monopolist (it will charge):

$$p^u = \arg\max (p^u - c) D^u(p^u)$$

If the upstream firm is a monopolist (it will charge):

$$\begin{aligned} p^u &= \arg\max (p^u - c) D^u(p^u) \\ FOC &: \frac{\partial (p^u - c) D(p^u)}{\partial p^u} = 0 \\ &\Longrightarrow (p^u - c) D'(p^u) = D(p^u) \end{aligned}$$

If the upstream firm is a monopolist (it will charge):

$$\begin{aligned} p^u &= \arg\max(p^u - c)D^u(p^u) \\ FOC &: \frac{\partial(p^u - c)D(p^u)}{\partial p^u} = 0 \\ &\implies (p^u - c)D'(p^u) = D(p^u) \\ &\implies p^u = -\frac{D(p^u)}{D'(p^u)} + c > c \\ \frac{p^u - c}{p^u} &= \frac{1}{\xi} = \frac{\alpha_i}{\xi} \end{aligned}$$

Where ξ is the elasticity of demand for the input (this is called the 'Lerner markup rule' where α_i represents the market share in Cournot competition, $\alpha_i = 1$ for monopoly).

Hence, for the retailer:

$$p_R^* = -\frac{D(p)}{D'(p)} - \frac{D(p^u)}{D'(p^u)} + c > p_I^*$$

Hence, for the retailer:

$$p_R^* = -\frac{D(p)}{D'(p)} - \frac{D(p^u)}{D'(p^u)} + c > p_I^*$$

Since the retailer also sets price and quantity so that MR=MC, but now his marginal cost is 'marked up' by $\frac{D(p^u)}{D'(p^u)}$ he will charge a higher price and sell a lower quantity than the integrated monopoly firm.

Since the integrated firm was both the upstream and downstream firm the integrated price p_I^* maximized the 'joint profit.' Hence, p_R^* must yield a lower 'joint profit'.

Double marginalization (linear example)

Assume linear demand (to consumers).

c : the constant marginal-cost production of the upstream firm

Double marginalization (linear example)

Assume linear demand (to consumers).

c : the constant marginal-cost production of the upstream firm

Downsteam firm's only cost is the input (one input per output), for which the upstream firm charges p_u.

$$\begin{aligned} p &= 1 - q \\ q^* &= \arg\max_q \pi_d = q(1 - q - p_u) \end{aligned}$$

Double marginalization (linear example)

Assume linear demand (to consumers).

c : the constant marginal-cost production of the upstream firm

Downsteam firm's only cost is the input (one input per output), for which the upstream firm charges p_u.

$$\begin{aligned} p &= 1 - q \\ q^* &= \arg\max_q \pi_d = q(1 - q - p_u) \\ \text{FOC} &: 1 - p_u - 2q^* = 0 \end{aligned}$$

Double marginalization (linear example)

Assume linear demand (to consumers).

c : the constant marginal-cost production of the upstream firm

Downsteam firm's only cost is the input (one input per output), for which the upstream firm charges p_u.

$$\begin{aligned} p &= 1 - q \\ q^* &= \arg\max_q \pi_d = q(1 - q - p_u) \\ \text{FOC} &: 1 - p_u - 2q^* = 0 \\ &\implies q^*(p_u) = \frac{(1 - p_u)}{2} \end{aligned}$$

Double marginalization (linear example)

Assume linear demand (to consumers).

c : the constant marginal-cost production of the upstream firm

Downsteam firm's only cost is the input (one input per output), for which the upstream firm charges p_u.

$$\begin{aligned}
p &= 1 - q \\
q^* &= \arg\max_q \pi_d = q(1 - q - p_u) \\
\text{FOC} &: 1 - p_u - 2q^* = 0 \\
&\implies q^*(p_u) = \frac{(1 - p_u)}{2} \\
p^*(p_u) &= 1 - \frac{(1 - p_u)}{2} = \frac{1 + p_u}{2}
\end{aligned}$$

Double marginalization (linear example)

Assume linear demand (to consumers).

c : the constant marginal-cost production of the upstream firm

Downsteam firm's only cost is the input (one input per output), for which the upstream firm charges p_u.

$$
\begin{aligned}
p &= 1 - q \\
q^* &= \arg\max_q \pi_d = q(1 - q - p_u) \\
\text{FOC} &: 1 - p_u - 2q^* = 0 \\
&\Longrightarrow q^*(p_u) = \frac{(1 - p_u)}{2} \\
p^*(p_u) &= 1 - \frac{(1 - p_u)}{2} = \frac{1 + p_u}{2} \\
&\Longrightarrow \pi_d^*(p_u) = (\frac{1 + p_u}{2} - p_u)(\frac{1 - p_u}{2}) \\
&= (\frac{1 - p_u}{2})^2
\end{aligned}
$$

The downstream firm will thus buy $\frac{(1-p_u)}{2}$ units of the input.
The upstream firm thus solves:

$$\max_{p_u}(p_u - c)q^*(p_u) \quad = \quad (p_u - c)\frac{1-p_u}{2}$$

$$\text{FOC} \quad : \quad \frac{1}{2}c - p_u + \frac{1}{2} = 0$$

The downstream firm will thus buy $\frac{(1-p_u)}{2}$ units of the input.
The upstream firm thus solves:

$$\begin{aligned}
\max_{p_u}(p_u - c)q^*(p_u) &= (p_u - c)\frac{1-p_u}{2} \\
\text{FOC} &: \frac{1}{2}c - p_u + \frac{1}{2} = 0 \\
&\implies p_u^* = \frac{1+c}{2}
\end{aligned}$$

The downstream firm will thus buy $\frac{(1-p_u)}{2}$ units of the input.
The upstream firm thus solves:

$$\max_{p_u}(p_u - c)q^*(p_u) = (p_u - c)\frac{1-p_u}{2}$$

$$\text{FOC} : \frac{1}{2}c - p_u + \frac{1}{2} = 0$$

$$\implies p_u^* = \frac{1+c}{2}$$

$$\implies \Pi_u^* = (\frac{1+c}{2} - c)\frac{1-\frac{1+c}{2}}{2} = \frac{(1-c)^2}{8}$$

The downstream firm will thus buy $\frac{(1-p_u)}{2}$ units of the input.
The upstream firm thus solves:

$$
\begin{aligned}
\max_{p_u}(p_u - c)q^*(p_u) &= (p_u - c)\frac{1-p_u}{2} \\
\text{FOC} &: \frac{1}{2}c - p_u + \frac{1}{2} = 0 \\
&\Longrightarrow p_u^* = \frac{1+c}{2} \\
&\Longrightarrow \Pi_u^* = (\frac{1+c}{2} - c)\frac{1-\frac{1+c}{2}}{2} = \frac{(1-c)^2}{8} \\
\Pi_u^* + \pi_d^*(p_u) &= \frac{(1-c)^2}{8} + (\frac{1-\frac{1+c}{2}}{2})^2 \\
&= \frac{3}{16}(1-c)^2
\end{aligned}
$$

An integrated firm would face marginal cost c, and thus solve:

$$\max_q \pi = q(1 - q - c)$$

An integrated firm would face marginal cost c, and thus solve:

$$\max_q \pi = q(1-q-c)$$
$$\implies \frac{\partial q(1-q-c)}{\partial q} = 1 - 2q - c = 0$$

An integrated firm would face marginal cost c, and thus solve:

$$\begin{aligned} \max_q \pi &= q(1-q-c) \\ &\Longrightarrow \frac{\partial q(1-q-c)}{\partial q} = 1-2q-c=0 \\ q^* &= \frac{1}{2}(1-c) \end{aligned}$$

An integrated firm would face marginal cost c, and thus solve:

$$\begin{aligned}
\max_q \pi &= q(1-q-c) \\
&\Longrightarrow \frac{\partial q(1-q-c)}{\partial q} = 1-2q-c = 0 \\
q^* &= \frac{1}{2}(1-c) \\
\pi^* &= \frac{1}{2}(1-c)(1-\frac{1}{2}(1-c)-c) \\
&= \frac{1}{4}(1-c)^2
\end{aligned}$$

An integrated firm would face marginal cost c, and thus solve:

$$\begin{aligned}
\max_q \pi &= q(1-q-c) \\
&\Longrightarrow \frac{\partial q(1-q-c)}{\partial q} = 1-2q-c=0 \\
q^* &= \frac{1}{2}(1-c) \\
\pi^* &= \frac{1}{2}(1-c)(1-\frac{1}{2}(1-c)-c) \\
&= \frac{1}{4}(1-c)^2 \\
&> \Pi_u^* + \pi_d^*(p_u) = \frac{3}{16}(1-c)^2
\end{aligned}$$

An integrated firm would face marginal cost c, and thus solve:

$$\begin{aligned}
\max_q \pi &= q(1-q-c) \\
&\Longrightarrow \frac{\partial q(1-q-c)}{\partial q} = 1-2q-c=0 \\
q^* &= \frac{1}{2}(1-c) \\
\pi^* &= \frac{1}{2}(1-c)(1-\frac{1}{2}(1-c)-c) \\
&= \frac{1}{4}(1-c)^2 \\
&> \Pi_u^* + \pi_d^*(p_u) = \frac{3}{16}(1-c)^2
\end{aligned}$$

So the integrated firm (and the consumer, as we could show a lower price) does better.

Note that an additional vertical layer is costly, but costs themselves (c) are only partially passed down.

Market power over outsiders.

Consider the case of Cournot (or Bertrand) duopolists. It is easy to see that if they set quantities independently then Nash equilibrium profits are (strictly) lower than the profits that a merged firm monopoly can make.

Market power over outsiders.

Consider the case of Cournot (or Bertrand) duopolists. It is easy to see that if they set quantities independently then Nash equilibrium profits are (strictly) lower than the profits that a merged firm monopoly can make.

To see this take a linear demand zero cost example and work out the duopoly output and profits and compare this to the monopoly profits.

Market power over outsiders.

Consider the case of Cournot (or Bertrand) duopolists. It is easy to see that if they set quantities independently then Nash equilibrium profits are (strictly) lower than the profits that a merged firm monopoly can make.

To see this take a linear demand zero cost example and work out the duopoly output and profits and compare this to the monopoly profits.

This suggests that the firms would be better off by merging, **if** collusion between firms is impossible (for legal reasons or because of failures of coordination).
...But we have firms even in very competitive industries – this can't be the only explanation for a firm.

Hidden Action or Information: Agency Theory and the Firm; I.e., what we studied in previous lectures

- Inefficient outcomes can result (under certain conditions) when the principal does not observe some action or attribute of the agent.

Hidden Action or Information: Agency Theory and the Firm; I.e., what we studied in previous lectures

- Inefficient outcomes can result (under certain conditions) when the principal does not observe some action or attribute of the agent.
- Is it easier (legally, technically) to observe information (e.g., cost, effort) within a firm then when contracting with another firm? If so, why?

Hidden Action or Information: Agency Theory and the Firm; I.e., what we studied in previous lectures

- Inefficient outcomes can result (under certain conditions) when the principal does not observe some action or attribute of the agent.
- Is it easier (legally, technically) to observe information (e.g., cost, effort) within a firm then when contracting with another firm? If so, why?
- Is it easier (legally, technically) to implement incentive schemes and mechanisms within a firm? If so, why?

Hidden Action or Information: Agency Theory and the Firm; I.e., what we studied in previous lectures

- Inefficient outcomes can result (under certain conditions) when the principal does not observe some action or attribute of the agent.
- Is it easier (legally, technically) to observe information (e.g., cost, effort) within a firm then when contracting with another firm? If so, why?
- Is it easier (legally, technically) to implement incentive schemes and mechanisms within a firm? If so, why?
- If so, this could explain the existence of firms, but not their scope (but see next slide).

Do agency costs increase in firm size?

Rem, from CEO lecture: Pay increases in firm size, but power of incentives decrease. Are increased agency problem is a cost of company size? Greater percentage ownership will discourage wasteful perquisites and vanity projects . . . But greater total value of shares owned will make the risk-aversion problem worse

Also, who "monitors the monitors"? Possible collusion.... (double markups in a sense?) See, eg, Strausz, 1997, "Delegation of monitoring in a Principal-Agent Relationship"

See also Lucas, 1978 "On the Size Distribution of Business Firms"

In 1919 General Motors decided to switch from open body cars to (present day) closed body cars. It approached Fisher Body a leading body producer to supply these new bodies. The production of these bodies required substantial investments on the part of Fisher Body. However, once these investments were in place it feared that General Motors may insist on only buying bodies at marginal costs, leading to large losses.

To provide proper incentives to Fisher Body, General Motors entered into a contract in which explicit provision was made for fixed costs and investment costs, by providing for a mark-up above variable costs. Faced with this contract, however, Fisher Body had an incentive to inflate its costs via overstaffing. General Motors had no option but to pay prices corresponding to the inflated costs, given the contracts. At the end, these costs proved to be too high and GM bought out Fisher Body. It is worth noting that the price of this buy-out was high since Fisher Body had a strong bargaining position based on high expected profits given the contract in place.

This case shows how a contract designed to stop GM from holding-up Fisher Body creates the possibility that Fisher Body can hold-up GM in turn.

This case shows how a contract designed to stop GM from holding-up Fisher Body creates the possibility that Fisher Body can hold-up GM in turn.

The problem of hold-up can be very serious. If an unforseen contingency arises; let's say some vehicle is invented that replaces the car, and GM needs to change the bodies of its cars slightly to make them military-ready, as the military is the only remaining demand. If the contract did not specify what should happen in such a contingency, Fisher would not be obligated to make this change to the body – it could claim this is too expensive. If GM would face virtually no demand with its current specifications, and it would take years to get another body producer equipped to work with GM, then Fisher would have a great deal of market power. Fisher could 'hold-up' GM for virtually all its expected profits from the new military vehicles.

This case shows how a contract designed to stop GM from holding-up Fisher Body creates the possibility that Fisher Body can hold-up GM in turn.

The problem of hold-up can be very serious. If an unforseen contingency arises; let's say some vehicle is invented that replaces the car, and GM needs to change the bodies of its cars slightly to make them military-ready, as the military is the only remaining demand. If the contract did not specify what should happen in such a contingency, Fisher would not be obligated to make this change to the body – it could claim this is too expensive. If GM would face virtually no demand with its current specifications, and it would take years to get another body producer equipped to work with GM, then Fisher would have a great deal of market power. Fisher could 'hold-up' GM for virtually all its expected profits from the new military vehicles.

Knowing that such unforseen contingencies are 'likely', GM will be reluctant to sign any contract commiting themselves to an outside firm (e.g., installing equipment) to manufacture its bodies. This can be modeled as in Grossman and Hart (1986).

Could the contract between GM and Fisher have been better written? The theory of ‘Incomplete Contracts’ claims that contracts cannot plan for every possible contingency.

Could the contract between GM and Fisher have been better written? The theory of ‘Incomplete Contracts’ claims that contracts cannot plan for every possible contingency.

Under integration the integrated firm has ‘ownership,’ and thus the right to make the decision what action to take if an unforseen contingency arises.

Could the contract between GM and Fisher have been better written? The theory of ‘Incomplete Contracts’ claims that contracts cannot plan for every possible contingency.

Under integration the integrated firm has ‘ownership,’ and thus the right to make the decision what action to take if an unforseen contingency arises.

But does vertical integration take care of all hold-up problems? Why should it matter?

Could the contract between GM and Fisher have been better written? The theory of 'Incomplete Contracts' claims that contracts cannot plan for every possible contingency.

Under integration the integrated firm has 'ownership,' and thus the right to make the decision what action to take if an unforseen contingency arises.

But does vertical integration take care of all hold-up problems? Why should it matter?

Does integration 'transform a hostile supplier into a docile employee'?

Transactions Costs/ "Rent Seeking"

- Earliest modern theories (Especially Williamson)

Transactions Costs/ "Rent Seeking"

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up

Transactions Costs/ “Rent Seeking”

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
    - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)

Transactions Costs/ "Rent Seeking"

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
  - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)
- 'Integration can stop socially destructive haggling over 'appropriable quasi-rents' (and optimal decision may not be achieved!)

Transactions Costs/ "Rent Seeking"

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
  - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)
- 'Integration can stop socially destructive haggling over 'appropriable quasi-rents' (and optimal decision may not be achieved!)
- Consider a 'war of attrition' over burning money

Transactions Costs/ “Rent Seeking”

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
  - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)
- ‘Integration can stop socially destructive haggling over ‘appropriable quasi-rents’ (and optimal decision may not be achieved!)
- Consider a ‘war of attrition’ over burning money
- Assumes decision by fiat under integration

Transactions Costs/ “Rent Seeking”

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
    - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)
- ‘Integration can stop socially destructive haggling over ‘appropriable quasi-rents’ (and optimal decision may not be achieved!)
- Consider a ‘war of attrition’ over burning money
- Assumes decision by fiat under integration
- GM-Fisher as classic case

Transactions Costs/ “Rent Seeking”

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
  - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)
- ‘Integration can stop socially destructive haggling over ‘appropriable quasi-rents’ (and optimal decision may not be achieved!)
- Consider a ‘war of attrition’ over burning money
- Assumes decision by fiat under integration
- GM-Fisher as classic case
- “Ownership can stop haggling that is undertaken via alienable instruments [e.g., physical capital]”

Transactions Costs/ “Rent Seeking”

- Earliest modern theories (Especially Williamson)
- Specific (but potentially contractible) investments and unforseen or uncontractible contingencies create opportunities for hold-up
  - Maybe we *can* forsee every contingency: Can we? (http://www.theonion.com/content/node/31051)
- ‘Integration can stop socially destructive haggling over ‘appropriable quasi-rents’ (and optimal decision may not be achieved!)
- Consider a ‘war of attrition’ over burning money
- Assumes decision by fiat under integration
- GM-Fisher as classic case
- “Ownership can stop haggling that is undertaken via alienable instruments [e.g., physical capital]”
- Original version of these theories are ‘incomplete’ – provide no explanation for firm’s scope – but Gibbons extends this.

Incentive-System Theory (E.g., Holstrom and Milgrom (1991))

`‘Asset ownership can be an instrument in a multi-task incentive problem’ (Gibbons)’`

Incentive-System Theory (E.g., Holstrom and Milgrom (1991))

`‘Asset ownership can be an instrument in a multi-task incentive problem’ (Gibbons)’`

Asset allocation affects incentives ... as in (later) Property Rights Model

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'
    - Can't write contract over its value (v)

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'
  - Can't write contract over its value (v)
- There is an output (y)and a 'performance measure' (P)

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'
  - Can't write contract over its value (v)
- There is an output (y)and a 'performance measure' (P)
- The output, the performance measure, and the asset are all potentially affected by:

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'
  - Can't write contract over its value (v)
- There is an output (y)and a 'performance measure' (P)
- The output, the performance measure, and the asset are all potentially affected by:
  1. The effort put into one task (a_1)

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'
  - Can't write contract over its value (v)
- There is an output (y)and a 'performance measure' (P)
- The output, the performance measure, and the asset are all potentially affected by:
  1. The effort put into one task (a_1)
  2. The effort put into another task (a_2) (or any of several tasks)

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an ‘instrument’
  - Can't write contract over its value (v)
- There is an output (y)and a ‘performance measure’ (P)
- The output, the performance measure, and the asset are all potentially affected by:
  1. The effort put into one task (a_1)
  2. The effort put into another task (a_2) (or any of several tasks)
  3. Random component (specific to each)

IS theory ...

Model follows Gibbons (2004)

- There is a **non-contractible asset** (e.g., a delivery truck, or a reputation for service) referred to as an 'instrument'
  - Can't write contract over its value (v)
- There is an output (y)and a 'performance measure' (P)
- The output, the performance measure, and the asset are all potentially affected by:
  1. The effort put into one task (a_1)
  2. The effort put into another task (a_2) (or any of several tasks)
  3. Random component (specific to each)
- Note, pay can be conditioned on the performance measure but not on output itself, perhaps output is observed only with great delay, is unverifiable, or is unobservable. Consider the education of a child..

Note: The matrix algebra is optional – you can solve this easily with simple algebra/calculus, which I also give.

$$y = f_1 a_1 + f_2 a_2 + \varepsilon = \mathbf{f}'\mathbf{a} + \varepsilon \qquad \text{(Production)}$$

Note: The matrix algebra is optional – you can solve this easily with simple algebra/calculus, which I also give.

$$y = f_1 a_1 + f_2 a_2 + \varepsilon = \mathbf{f}'\mathbf{a} + \varepsilon \qquad \text{(Production)}$$

$$P = g_1 a_1 + g_2 a_2 + \phi = \mathbf{g}'\mathbf{a} + \phi \qquad \text{(Performance Measure)}$$

Note: The matrix algebra is optional – you can solve this easily with simple algebra/calculus, which I also give.

$$y = f_1 a_1 + f_2 a_2 + \varepsilon = \mathbf{f}'\mathbf{a} + \varepsilon \qquad \text{(Production)}$$

$$P = g_1 a_1 + g_2 a_2 + \phi = \mathbf{g}'\mathbf{a} + \phi \qquad \text{(Performance Measure)}$$

$$v = h_1 a_1 + h_2 a_2 + \xi = \mathbf{h}'\mathbf{a} + \xi \qquad \text{(Asset)}$$

Payoffs (other than asset)

$$
\begin{aligned}
U_{p(rincipal)} &= y - w \\
U_{A(gent)} &= w - c(a_1, a_2)
\end{aligned}
$$

Payoffs (other than asset)

$$
\begin{aligned}
U_{p(rincipal)} &= y - w \\
U_{A(gent)} &= w - c(a_1, a_2)
\end{aligned}
$$

Cost function:

$$c(a_1, a_2) = \frac{1}{2}(a_1^2 + a_2^2) = \frac{1}{2}\mathbf{a}'\mathbf{a}$$

Payoffs (other than asset)

$$
\begin{aligned}
U_{p(rincipal)} &= y - w \\
U_{A(gent)} &= w - c(a_1, a_2)
\end{aligned}
$$

Cost function:

$$c(a_1, a_2) = \frac{1}{2}(a_1^2 + a_2^2) = \frac{1}{2}\mathbf{a}'\mathbf{a}$$

Linear wage:

$$w = S + bP$$

Note: Ignoring the asset/instrument, the efficient "slope" of pay in the performance measure will depend on the relation between of the performance measure and the effort and output – i.e., the "alignment" of the **f** and **g** vectors.

Payoffs (other than asset)

$$U_{p(rincipal)} = y - w$$
$$U_{A(gent)} = w - c(a_1, a_2)$$

Cost function:

$$c(a_1, a_2) = \frac{1}{2}(a_1^2 + a_2^2) = \frac{1}{2}\mathbf{a}'\mathbf{a}$$

Linear wage:

$$w = S + bP$$

Note: Ignoring the asset/instrument, the efficient "slope" of pay in the performance measure will depend on the relation between of the performance measure and the effort and output – i.e., the "alignment" of the **f** and **g** vectors.

First-best actions (vector):

$$
\begin{aligned}
\mathbf{a}^{FB} &= \mathbf{f} + \mathbf{h} \\
a_1^{FB} &= f_1 + h_1 \\
a_2^{FB} &= f_2 + h_2
\end{aligned}
$$

First-best actions (vector):

$$\begin{aligned} \mathbf{a}^{FB} &= \mathbf{f} + \mathbf{h} \\ a_1^{FB} &= f_1 + h_1 \\ a_2^{FB} &= f_2 + h_2 \end{aligned}$$

Note: Even though there is a potentially unobserved action, the Principal can capture the entire generated surplus because he has all the bargaining power and there is no risk-aversion nor credit constraints. Thus, he always wants to maximize $y + v$.

Notes on matrix derivatives:

Imagine a vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Notes on matrix derivatives:

Imagine a vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Define: $\partial \mathbf{X}(b) = \partial_b \mathbf{X} = \begin{bmatrix} \frac{\partial X_1}{\partial b} \\ \frac{\partial X_2}{\partial b} \end{bmatrix}$

Notes on matrix derivatives:

Imagine a vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Define: $\partial \mathbf{X}(b) = \partial_b \mathbf{X} = \begin{bmatrix} \frac{\partial X_1}{\partial b} \\ \frac{\partial X_2}{\partial b} \end{bmatrix}$

Note: $\partial_b \left(b\mathbf{g} + \mathbf{h} \right) = \mathbf{g}$

Notes on matrix derivatives:

Imagine a vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Define: $\partial\mathbf{X}(b) = \partial_b \mathbf{X} = \begin{bmatrix} \frac{\partial X_1}{\partial b} \\ \frac{\partial X_2}{\partial b} \end{bmatrix}$

Note: $\partial_b \left(b\mathbf{g} + \mathbf{h} \right) = \mathbf{g}$

Note: $\frac{1}{2}\mathbf{X}(b)'\mathbf{X}(b) = \frac{1}{2}(X_1(b)^2 + X_2(b)^2)$

Notes on matrix derivatives:

Imagine a vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Define: $\partial \mathbf{X}(b) = \partial_b \mathbf{X} = \begin{bmatrix} \frac{\partial X_1}{\partial b} \\ \frac{\partial X_2}{\partial b} \end{bmatrix}$

Note: $\partial_b \left(b\mathbf{g} + \mathbf{h} \right) = \mathbf{g}$

Note: $\frac{1}{2}\mathbf{X}(b)'\mathbf{X}(b) = \frac{1}{2}(X_1(b)^2 + X_2(b)^2)$
$\Rightarrow \frac{\partial}{\partial b} \left(\frac{1}{2}\mathbf{X}(b)'\mathbf{X}(b) \right) = X_1 \frac{\partial X_1}{\partial b} + X_2 \frac{\partial X_2}{\partial b} = \mathbf{X}'\partial_b \mathbf{X}$

Notes on matrix derivatives:

Imagine a vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

Define: $\partial \mathbf{X}(b) = \partial_b \mathbf{X} = \begin{bmatrix} \frac{\partial X_1}{\partial b} \\ \frac{\partial X_2}{\partial b} \end{bmatrix}$

Note: $\partial_b \left(b\mathbf{g} + \mathbf{h} \right) = \mathbf{g}$

Note: $\frac{1}{2}\mathbf{X}(b)'\mathbf{X}(b) = \frac{1}{2}(X_1(b)^2 + X_2(b)^2)$
$\Rightarrow \frac{\partial}{\partial b} \left(\frac{1}{2}\mathbf{X}(b)'\mathbf{X}(b) \right) = X_1 \frac{\partial X_1}{\partial b} + X_2 \frac{\partial X_2}{\partial b} = \mathbf{X}'\partial_b \mathbf{X}$

Note: this notation is nonstandard. Economists usually use a subscript to refer to a vector derivative, but the subscript was already in use here.

Case 1: Principal owns asset, Agent an 'Employee'

Case 1: Principal owns asset, Agent an 'Employee'

$$U_p = y + v - w$$
$$U_A = w - c$$

Induced actions (of $w = S + bP$)**:**

Case 1: Principal owns asset, Agent an 'Employee'

$$U_p = y + v - w$$
$$U_A = w - c$$

Induced actions (of $w = S + bP$):

$$\mathbf{a}_E^*(b) = b\mathbf{g}$$
$$i.e,\ a_1^*(b) = bg_1,\ a_2^*(b) = bg_2$$

(i.e., this is the employee's maximizing action given this wage, when P owns the asset)

Case 1: Principal owns asset, Agent an 'Employee'

$$U_p = y + v - w$$
$$U_A = w - c$$

Induced actions (of $w = S + bP$)**:**

$$\mathbf{a}_E^*(b) = b\mathbf{g}$$
$$i.e,\ a_1^*(b) = bg_1,\ a_2^*(b) = bg_2$$

(i.e., this is the employee's maximizing action given this wage, when P owns the asset)
b_E^* set to maximize expected total surplus, since the principal can capture the total surplus with a forcing contract:

Case 1: Principal owns asset, Agent an 'Employee'

$$U_p = y + v - w$$
$$U_A = w - c$$

Induced actions (of $w = S + bP$):

$$\mathbf{a}_E^*(b) = b\mathbf{g}$$
$$i.e,\ a_1^*(b) = bg_1,\ a_2^*(b) = bg_2$$

(i.e., this is the employee's maximizing action given this wage, when P owns the asset) b_E^* set to maximize expected total surplus, since the principal can capture the total surplus with a forcing contract:

$$TS_E(b) = E(y + v) - c(a_1, a_2)$$

Case 1: Principal owns asset, Agent an 'Employee'

$$\begin{aligned} U_p &= y + v - w \\ U_A &= w - c \end{aligned}$$

Induced actions (of $w = S + bP$)**:**

$$\begin{aligned} \mathbf{a}_E^*(b) &= b\mathbf{g} \\ i.e,\ a_1^*(b) &= bg_1,\ a_2^*(b) = bg_2 \end{aligned}$$

(i.e., this is the employee's maximizing action given this wage, when P owns the asset)
b_E^* set to maximize expected total surplus, since the principal can capture the total surplus with a forcing contract:

$$\begin{aligned} TS_E(b) &= E(y+v) - c(a_1, a_2) \\ &= (\mathbf{f}+\mathbf{h})'\mathbf{a}_E^*(b) - \frac{1}{2}\mathbf{a}_E^*(b)'\mathbf{a}_E^*(b) \\ &= (f_1+h_1)a_{1E}^*(b) + (f_2+h_2)a_{2E}^*(b) - \frac{1}{2}a_{1E}^*(b)^2 - \frac{1}{2}a_{2E}^*(b)^2 \end{aligned}$$

Case 2: Agent owns asset, Agent a 'Contractor'

$$
\begin{aligned}
U_p &= y - w \\
U_A &= w + v - c
\end{aligned}
$$

Case 2: Agent owns asset, Agent a 'Contractor'

$$U_p = y - w$$
$$U_A = w + v - c$$

Induced actions (of $w = S + bP$)**:**

$$\mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h}$$

Case 2: Agent owns asset, Agent a ‘Contractor’

$$U_p = y - w$$
$$U_A = w + v - c$$

Induced actions (of $w = S + bP$)**:**

$$\mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h}$$

b_c^* set to maximize expected total surplus, since the principal can capture this:

$$\begin{aligned} TS_C(b) &= (\mathbf{f} + \mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b) \\ &= (f_1 + h_1)a_{1C}^*(b) + (f_2 + h_2)a_{2C}^*(b) - \frac{1}{2}a_{1C}^*(b)^2 - \frac{1}{2}a_{2C}^*(b)^2 \end{aligned}$$

(Note: this expression for total surplus is the same as in case 1. The difference is the agent's response to a contract $a_C^*(b)$ versus $a_E^*(b)$)

Case 2: Agent owns asset, Agent a 'Contractor'

$$\begin{aligned} U_p &= y - w \\ U_A &= w + v - c \end{aligned}$$

Induced actions (of $w = S + bP$):

$$\mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h}$$

b_c^* set to maximize expected total surplus, since the principal can capture this:

$$\begin{aligned} TS_C(b) &= (\mathbf{f} + \mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b) \\ &= (f_1 + h_1)a_{1C}^*(b) + (f_2 + h_2)a_{2C}^*(b) - \frac{1}{2}a_{1C}^*(b)^2 - \frac{1}{2}a_{2C}^*(b)^2 \end{aligned}$$

(Note: this expression for total surplus is the same as in case 1. The difference is the agent's response to a contract $a_C^*(b)$ versus $a_E^*(b)$)

Issue: which type of ownership allows greater total surplus?

Case 2: Agent owns asset, Agent a 'Contractor'

$$U_p = y - w$$
$$U_A = w + v - c$$

Induced actions (of $w = S + bP$)**:**

$$\mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h}$$

b_c^* set to maximize expected total surplus, since the principal can capture this:

$$TS_C(b) = (\mathbf{f} + \mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b)$$
$$= (f_1 + h_1)a_{1C}^*(b) + (f_2 + h_2)a_{2C}^*(b) - \frac{1}{2}a_{1C}^*(b)^2 - \frac{1}{2}a_{2C}^*(b)^2$$

(Note: this expression for total surplus is the same as in case 1. The difference is the agent's response to a contract $a_C^*(b)$ versus $a_E^*(b)$)

Issue: which type of ownership allows greater total surplus?

Answer: it depends on the parameters.

Extreme example 1:

$$E(y) = a_1, \ E(v) = a_2, \ E(P) = a_1 + a_2$$

Extreme example 1:

$$
\begin{aligned}
E(y) &= a_1,\ E(v) = a_2,\ E(P) = a_1 + a_2 \\
\text{i.e., } \mathbf{f} &= \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

- One action impacts the production the other impacts the asset.

Extreme example 1:

$$E(y) = a_1,\ E(v) = a_2,\ E(P) = a_1 + a_2$$
$$\text{i.e., } \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- One action impacts the production the other impacts the asset.
- The sum of these actions is observed

Extreme example 1:

$$\begin{aligned} E(y) &= a_1,\ E(v) = a_2,\ E(P) = a_1 + a_2 \\ \text{i.e., } \mathbf{f} &= \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

- One action impacts the production the other impacts the asset.
- The sum of these actions is observed
- $\mathbf{g} = \mathbf{f} + \mathbf{h}$ ... 'P is perfectly aligned with y and v.'

If principal owns asset and hires employee:

$$\mathbf{a}_E^*(b) = b\mathbf{g} = \begin{bmatrix} b \\ b \end{bmatrix}$$
$$a_{1E}^* = bg_1, a_{2E}^* = bg_2$$

If principal owns asset and hires employee:

$$\begin{aligned} \mathbf{a}_E^*(b) &= b\mathbf{g} = \begin{bmatrix} b \\ b \end{bmatrix} \\ a_{1E}^* &= bg_1, a_{2E}^* = bg_2 \end{aligned}$$

Setting $b = 1$:

$$\begin{aligned} \mathbf{a}_E^*(b) &= b\mathbf{g} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \mathbf{f} + \mathbf{h} = \mathbf{a}^{FB} \\ a_{1E}^* &= bg_1 = b = 1 = f_1 + h_1 = 1 + 0 = a_1^{FB} \\ a_{2E}^* &= bg_2 = b = 1 = f_2 + h_2 = 0 + 1 = a_2^{FB} \end{aligned}$$

So, the optimum can be attained in this case if P owns the asset.

Extreme example 1 continued

Suboptimal ownership

On the other hand, if the agent owns the asset (as a 'contractor'):
Would choose b to maximize:

$$TS_C(b) = E(y+v) - c(a_1, a_2)$$

Extreme example 1 continued

Suboptimal ownership

On the other hand, if the agent owns the asset (as a 'contractor'):
Would choose b to maximize:

$$\begin{aligned} TS_C(b) &= E(y+v) - c(a_1, a_2) \\ &= (\mathbf{f}+\mathbf{h})'\mathbf{a}^*_C(b) - \frac{1}{2}\mathbf{a}^*_C(b)'\mathbf{a}^*_C(b) \\ &= (f_1+h_1)a^*_{1C}(b) + (f_2+h_2)a^*_{2C}(b) - \frac{1}{2}a^*_{1C}(b)^2 - \frac{1}{2}a^*_{2C}(b)^2 \end{aligned}$$

Taking the first-order conditions (using vectors; do it without matrix algebra as an exercise):

$$\frac{\partial}{\partial b} TS_C(b) \quad = \quad \frac{\partial}{\partial b}\left((\mathbf{f}+\mathbf{h})'\mathbf{a}^*_C(b) - \frac{1}{2}\mathbf{a}^*_C(b)'\mathbf{a}^*_C(b) \right) = 0$$

Taking the first-order conditions (using vectors; do it without matrix algebra as an exercise):

$$\begin{aligned}\frac{\partial}{\partial b}TS_C(b) &= \frac{\partial}{\partial b}\left((\mathbf{f}+\mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b)\right) = 0 \\ &\Longrightarrow (\mathbf{f}+\mathbf{h})'\partial\mathbf{a}_C^*(b) - \mathbf{a}_C^*(b)'\partial\mathbf{a}_C^*(b) = 0 \\ &= (\mathbf{f}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) - (b\mathbf{g}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h})\end{aligned}$$

Taking the first-order conditions (using vectors; do it without matrix algebra as an exercise):

$$
\begin{aligned}
\frac{\partial}{\partial b} TS_C(b) &= \frac{\partial}{\partial b}\left((\mathbf{f}+\mathbf{h})' \mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)' \mathbf{a}_C^*(b) \right) = 0 \\
&\Longrightarrow (\mathbf{f}+\mathbf{h})' \partial \mathbf{a}_C^*(b) - \mathbf{a}_C^*(b)' \partial \mathbf{a}_C^*(b) = 0 \\
&= (\mathbf{f}+\mathbf{h})' \partial (b\mathbf{g}+\mathbf{h}) - (b\mathbf{g}+\mathbf{h})' \partial (b\mathbf{g}+\mathbf{h}) \\
&= (\mathbf{f}+\mathbf{h})' \mathbf{g} - (b\mathbf{g}+\mathbf{h})' \mathbf{g} = (\mathbf{f}-b\mathbf{g})' \mathbf{g}
\end{aligned}
$$

Taking the first-order conditions (using vectors; do it without matrix algebra as an exercise):

$$
\begin{aligned}
\frac{\partial}{\partial b} TS_C(b) &= \frac{\partial}{\partial b}\left((\mathbf{f}+\mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b)\right) = 0 \\
&\Longrightarrow (\mathbf{f}+\mathbf{h})'\partial\mathbf{a}_C^*(b) - \mathbf{a}_C^*(b)'\partial\mathbf{a}_C^*(b) = 0 \\
&= (\mathbf{f}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) - (b\mathbf{g}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) \\
&= (\mathbf{f}+\mathbf{h})'\mathbf{g} - (b\mathbf{g}+\mathbf{h})'\mathbf{g} = (\mathbf{f}-b\mathbf{g})'\mathbf{g} \\
&= \begin{bmatrix} 1-b \\ -b \end{bmatrix}' \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -2b+1 = 0 \\
&\Longrightarrow b_C^* = \frac{1}{2}
\end{aligned}
$$

Taking the first-order conditions (using vectors; do it without matrix algebra as an exercise):

$$
\begin{aligned}
\frac{\partial}{\partial b} TS_C(b) &= \frac{\partial}{\partial b}\left((\mathbf{f}+\mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b) \right) = 0 \\
&\Longrightarrow (\mathbf{f}+\mathbf{h})'\partial\mathbf{a}_C^*(b) - \mathbf{a}_C^*(b)'\partial\mathbf{a}_C^*(b) = 0 \\
&= (\mathbf{f}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) - (b\mathbf{g}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) \\
&= (\mathbf{f}+\mathbf{h})'\mathbf{g} - (b\mathbf{g}+\mathbf{h})'\mathbf{g} = (\mathbf{f}-b\mathbf{g})'\mathbf{g} \\
&= \begin{bmatrix} 1-b \\ -b \end{bmatrix}' \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -2b+1 = 0 \\
&\Longrightarrow b_C^* = \frac{1}{2} \\
&\Longrightarrow \mathbf{a}_C^*(b) = b\mathbf{g}+\mathbf{h} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}
\end{aligned}
$$

Taking the first-order conditions (using vectors; do it without matrix algebra as an exercise):

$$
\begin{aligned}
\frac{\partial}{\partial b} TS_C(b) &= \frac{\partial}{\partial b}\left((\mathbf{f}+\mathbf{h})'\mathbf{a}_C^*(b) - \frac{1}{2}\mathbf{a}_C^*(b)'\mathbf{a}_C^*(b) \right) = 0 \\
&\Longrightarrow (\mathbf{f}+\mathbf{h})'\partial\mathbf{a}_C^*(b) - \mathbf{a}_C^*(b)'\partial\mathbf{a}_C^*(b) = 0 \\
&= (\mathbf{f}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) - (b\mathbf{g}+\mathbf{h})'\partial(b\mathbf{g}+\mathbf{h}) \\
&= (\mathbf{f}+\mathbf{h})'\mathbf{g} - (b\mathbf{g}+\mathbf{h})'\mathbf{g} = (\mathbf{f}-b\mathbf{g})'\mathbf{g} \\
&= \begin{bmatrix} 1-b \\ -b \end{bmatrix}' \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -2b+1 = 0 \\
&\Longrightarrow b_C^* = \frac{1}{2} \\
&\Longrightarrow \mathbf{a}_C^*(b) = b\mathbf{g}+\mathbf{h} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix} \\
&\neq \mathbf{a}^{FB} = \mathbf{f}+\mathbf{h} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned}
$$

$$\implies \quad \mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$$

$$\begin{aligned} \implies \quad & \mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix} \\ \neq \quad & \mathbf{a}^{FB} = \mathbf{f} + \mathbf{h} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

Intuition:

- Since $g = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, no way to motivate a_1 any more or less than a_2; increase in b increases both equally.

$$\implies \mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h} = \frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\\frac{3}{2}\end{bmatrix}$$

$$\neq \mathbf{a}^{FB} = \mathbf{f} + \mathbf{h} = \begin{bmatrix}1\\1\end{bmatrix}$$

Intuition:

- Since $g = \begin{bmatrix}1\\1\end{bmatrix}$, no way to motivate a_1 any more or less than a_2; increase in b increases both equally.
- ...But principal needs to do this, to balance out incentives (to not 'overinvest' in asset), since the agent already gets 'private' reward from asset.

$$\implies \mathbf{a}_C^*(b) = b\mathbf{g} + \mathbf{h} = \frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\\frac{3}{2}\end{bmatrix}$$

$$\neq \mathbf{a}^{FB} = \mathbf{f} + \mathbf{h} = \begin{bmatrix}1\\1\end{bmatrix}$$

Intuition:

- Since $g = \begin{bmatrix}1\\1\end{bmatrix}$, no way to motivate a_1 any more or less than a_2; increase in b increases both equally.
- ...But principal needs to do this, to balance out incentives (to not 'overinvest' in asset), since the agent already gets 'private' reward from asset.
- So agent over-exerts in a_2, the 'asset builder', and under-exerts in a_1, the 'output builder'

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

One action affects output, the other affects the asset, but only a_1 (the output-shifter) observed

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

One action affects output, the other affects the asset, but only a_1 (the output-shifter) observed

'P aligned with y'

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

One action affects output, the other affects the asset, but only a_1 (the output-shifter) observed

'P aligned with y'

Can induce first-best if agent owns machine (contractor). Otherwise she may not care for the machine.

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

One action affects output, the other affects the asset, but only a_1 (the output-shifter) observed

'P aligned with y'

Can induce first-best if agent owns machine (contractor). Otherwise she may not care for the machine.

$$\begin{aligned} \implies \quad & \mathbf{a}_c^*(b) = b\mathbf{g} + \mathbf{h} \\ = \quad & \begin{bmatrix} b \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

One action affects output, the other affects the asset, but only a_1 (the output-shifter) observed

'P aligned with y'

Can induce first-best if agent owns machine (contractor). Otherwise she may not care for the machine.

$$\begin{aligned} \Longrightarrow \quad & \mathbf{a}_c^*(b) = b\mathbf{g} + \mathbf{h} \\ = \quad & \begin{bmatrix} b \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

Extreme Example 2

$$E(y) = a_1, E(v) = a_2, E(p) = a_1$$

I.e.

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{h} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

One action affects output, the other affects the asset, but only a_1 (the output-shifter) observed

'P aligned with y'

Can induce first-best if agent owns machine (contractor). Otherwise she may not care for the machine.

$$\begin{aligned} & \Longrightarrow & \mathbf{a}_c^*(b) = b\mathbf{g} + \mathbf{h} \\ & = & \begin{bmatrix} b \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \text{Setting } b_c^* & = & 1 \\ & \Longrightarrow & \mathbf{a}_\mathbf{c}^*(b_c^*) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \mathbf{a}^{FB} \end{aligned}$$

Conclusion: "asset ownership is useful when it gives the Principal improved control over the Agent's incentives."

Crossing over? Are more "distant" (more layers of authority) assets and employees less likely to be observable in their "preservation" activities?

Exercise: What if principal owns the machine (in above example)?

Adaptation

Simon (1951; employment context); model follows Gibbons (2004)

Two parties: Boss (B), Employee (E)

Can either:

Adaptation

Simon (1951; employment context); model follows Gibbons (2004)

Two parties: Boss (B), Employee (E)
Can either:
(a) Negotiate decision $d \in D$ before uncertainty ($s \in S$) is resolved
Or

Adaptation

Simon (1951; employment context); model follows Gibbons (2004)

Two parties: Boss (B), Employee (E)
Can either:
(a) Negotiate decision $d \in D$ before uncertainty ($s \in S$) is resolved
Or
(b) Allocate authority to decide on d after s is observed

Payoffs:
$U_B(s, d)$ and $U_E(s, d)$ $^{+}/_{-}$ transfers. Note these utility functions may not be the same, but both may depend on s!

(a) Ex-ante negotiation (assume efficient negotiation)

$$d^* = \arg\max_{d \in D} E_s \left[U_B(s, d) + U_E(s, d) \right]$$

(a) Ex-ante negotiation (assume efficient negotiation)

$$d^* = \arg\max_{d \in D} E_s \left[U_B(s,d) + U_E(s,d) \right]$$

(b) Authority to boss

$$d_B(s) = \arg\max_{d \in D} U_B(s,d)$$

(a) Ex-ante negotiation (assume efficient negotiation)

$$d^* = \arg\max_{d \in D} E_s \left[U_B(s, d) + U_E(s, d) \right]$$

(b) Authority to boss

$$d_B(s) = \arg\max_{d \in D} U_B(s, d)$$

If $E_s \left[U_B(s, d_B(s)) + U_E(s, d_B(s)) \right] > E_s \left[U_B(s, d^*) + U_E(s, d^*) \right]$ then (b) Pareto-dominates (a). If the inequalities are reversed, then (a) Pareto-dominates (b).

(a) Ex-ante negotiation (assume efficient negotiation)

$$d^* = \arg\max_{d \in D} E_s \left[U_B(s,d) + U_E(s,d) \right]$$

(b) Authority to boss

$$d_B(s) = \arg\max_{d \in D} U_B(s,d)$$

If $E_s \left[U_B(s, d_B(s)) + U_E(s, d_B(s)) \right] > E_s \left[U_B(s, d^*) + U_E(s, d^*) \right]$ then (b) Pareto-dominates (a). If the inequalities are reversed, then (a) Pareto-dominates (b).

(c) Authority to employee $\Longrightarrow$

$$d_E(s) = \arg\max_{d \in D} U_E(s,d)$$

Why ‘Pareto-dominates’? More total surplus is generated.

If we are in the ownership structure that is Pareto-dominated, one side could make the other side an offer to switch to the other regime and compensate them for their different utility, and still have enough surplus left over to do better. So one does better without making the other worse off.

Analogy to integration and make/buy issues:

Analogy to integration and make/buy issues:
a) Ex-ante negotiation: Like a very strict locked-in contract between firms, or between a principal and agent within a firm.

b) Authority to boss: Like integration; manager will act in own interest, 'hold-up' (e.g.) upstream division if desired

c) Authority to employee: Like non-integration, no locked in contract, flexible dealings with outside (e.g.) supplier

Analogies to life:

- Give your step-brother the keys to the house and car while you are away?

Analogies to life:

- Give your step-brother the keys to the house and car while you are away?
- Give your employees strict rules or free-reign over project?

Analogies to life:

- Give your step-brother the keys to the house and car while you are away?
- Give your employees strict rules or free-reign over project?
- Give the fox keys to the henhouse in case of a flood or fire?

Analogies to life:

- Give your step-brother the keys to the house and car while you are away?
- Give your employees strict rules or free-reign over project?
- Give the fox keys to the henhouse in case of a flood or fire?
  - What if both the principal and agent are foxes?

Analogies to life:

- Give your step-brother the keys to the house and car while you are away?
- Give your employees strict rules or free-reign over project?
- Give the fox keys to the henhouse in case of a flood or fire?
    - What if both the principal and agent are foxes?
- Give your spouse control of your investments in case you are incapacitated?

First key difference from Property-Rights model: decisions (d) not contractible ex-post – only decision rights.
(so we may have an inefficient decision)

First key difference from Property-Rights model: decisions (d) not contractible ex-post – only decision rights.
(so we may have an inefficient decision)

How to justify this:

- Need quick decision

First key difference from Property-Rights model: decisions (d) not contractible ex-post – only decision rights.
(so we may have an inefficient decision)

How to justify this:

- Need quick decision
- Moral hazard (unobservable/unverifiable action) ex-post

First key difference from Property-Rights model: decisions (d) not contractible ex-post – only decision rights.
(so we may have an inefficient decision)

How to justify this:

- Need quick decision
- Moral hazard (unobservable/unverifiable action) ex-post
- Costs of communication in the firm

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$
3. Party with control chooses $d \in D$

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$
3. Party with control chooses $d \in D$
4. Payoffs $U_i(S, d), i = 1, 2$

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$
3. Party with control chooses $d \in D$
4. Payoffs $U_i(S, d), i = 1, 2$

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$
3. Party with control chooses $d \in D$
4. Payoffs $U_i(S, d), i = 1, 2$

$\Longrightarrow$When i has control:

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$
3. Party with control chooses $d \in D$
4. Payoffs $U_i(S, d), i = 1, 2$

$\Longrightarrow$When i has control:

$$TS^i = E_s \left[U_1(s, d^i(s)) + U_2(s, d^i(s)) \right]$$

Adaptation adapted to the firm: Baker, Gibbons, and Murphy (2004)

Basically the same thing.

Can't renegotiate decisions nor decision rights ex-post; decisions unverifiable.

Timing:

1. Negotiate decision right
2. Both parties observe $s \in S$, distributed with probability $P(s)$
3. Party with control chooses $d \in D$
4. Payoffs $U_i(S, d), i = 1, 2$

$\Longrightarrow$When i has control:

$$TS^i = E_s \left[U_1(s, d^i(s)) + U_2(s, d^i(s)) \right]$$

$\Longrightarrow$Will give control to the party whose ex-post incentives are more in line with maximizing total surplus.
(assumed preferable to strict ex-ante contract)

Comments on adaptation:

- No specific investments are required

Comments on adaptation:

- No specific investments are required
- Like property-rights in that asset ownership determines (only) decision rights

Comments on adaptation:

- No specific investments are required
- Like property-rights in that asset ownership determines (only) decision rights
  - But here decisions are not contractible ex-post

Comments on adaptation:

- No specific investments are required
- Like property-rights in that asset ownership determines (only) decision rights
  - But here decisions are not contractible ex-post
- Unlike Incentive System, assets have no direct impact on payoffs

Comments on adaptation:

- No specific investments are required
- Like property-rights in that asset ownership determines (only) decision rights
  - But here decisions are not contractible ex-post
- Unlike Incentive System, assets have no direct impact on payoffs
- Crossing over?

Comments on adaptation:

- No specific investments are required
- Like property-rights in that asset ownership determines (only) decision rights
  - But here decisions are not contractible ex-post
- Unlike Incentive System, assets have no direct impact on payoffs
- Crossing over?
  - With more layers of authority and more distant decisions, do the agent's preferences tend to be more in line with maximizing total surplus?

Comments on adaptation:

- No specific investments are required
- Like property-rights in that asset ownership determines (only) decision rights
  - But here decisions are not contractible ex-post
- Unlike Incentive System, assets have no direct impact on payoffs
- Crossing over?
  - With more layers of authority and more distant decisions, do the agent's preferences tend to be more in line with maximizing total surplus?
  - Extension: What if the ex-post information observed is different for principal and agent?

Property Rights: The Grossman and Hart model

- Ownership as residual rights

Property Rights: The Grossman and Hart model

- Ownership as residual rights
- Incomplete contracts

Property Rights: The Grossman and Hart model

- Ownership as residual rights
- Incomplete contracts
- Hold-up: opportunistic behavior

Property Rights: The Grossman and Hart model

- Ownership as residual rights
- Incomplete contracts
- Hold-up: opportunistic behavior
- Ex-post rent extraction leads to ex-ante distortions

Property Rights: The Grossman and Hart model

- Ownership as residual rights
- Incomplete contracts
- Hold-up: opportunistic behavior
- Ex-post rent extraction leads to ex-ante distortions
- Ownership structured to minimize these distortions

Property Rights: The Grossman and Hart model

- Ownership as residual rights
- Incomplete contracts
- Hold-up: opportunistic behavior
- Ex-post rent extraction leads to ex-ante distortions
- Ownership structured to minimize these distortions
- Offers an explanation for the scope of firms as well as their existence.

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)
- "Level playing field": "evaluates life under integration ... for the same environment [as] under non-integration" (integration not a "magic pill")

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)
- "Level playing field": "evaluates life under integration ... for the same environment [as] under non-integration" (integration not a "magic pill")
- Distinction: "Where the rent seeking theory envisions socially destructive haggling ex post, the property-rights theory assumes efficient bargaining ... [and] requires non-contractible specific investments"

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)
- "Level playing field": "evaluates life under integration ... for the same environment [as] under non-integration" (integration not a "magic pill")
- Distinction: "Where the rent seeking theory envisions socially destructive haggling ex post, the property-rights theory assumes efficient bargaining ... [and] requires non-contractible specific investments"

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)
- "Level playing field": "evaluates life under integration ... for the same environment [as] under non-integration" (integration not a "magic pill")
- Distinction: "Where the rent seeking theory envisions socially destructive haggling ex post, the property-rights theory assumes efficient bargaining ... [and] requires non-contractible specific investments"

"Each party's surplus share determines that party's investment incentive ...

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)
- "Level playing field": "evaluates life under integration ... for the same environment [as] under non-integration" (integration not a "magic pill")
- Distinction: "Where the rent seeking theory envisions socially destructive haggling ex post, the property-rights theory assumes efficient bargaining ... [and] requires non-contractible specific investments"

"Each party's surplus share determines that party's investment incentive ...
...ownership determines that party's surplus share ...

Property rights, relation-specific investments, incomplete contracts

- Earliest complete and unified formal theory of the firm (Grossman and Hart, 1986)
- "Level playing field": "evaluates life under integration ... for the same environment [as] under non-integration" (integration not a "magic pill")
- Distinction: "Where the rent seeking theory envisions socially destructive haggling ex post, the property-rights theory assumes efficient bargaining ... [and] requires non-contractible specific investments"

"Each party's surplus share determines that party's investment incentive ...
...ownership determines that party's surplus share ...
...if it is important to maximize one party's investment then that party should own all the assets"

$\Rightarrow$ "Using a formal instrument to stop one hold-up problem typically creates another hold-up problem"...

⇒ "Using a formal instrument to stop one hold-up problem typically creates another hold-up problem"...

"The cost of control is the loss of initiative"

⇒ "Using a formal instrument to stop one hold-up problem typically creates another hold-up problem"...

"The cost of control is the loss of initiative"

⇒!!A downside of integration!!

- Weakness of (PR) theory: Reverse extreme as under Rent Seeking – Employees are now assumed to have no incentives to work, nor can they be motivated through managers using 'mechanisms' to do so.

⇒ "Using a formal instrument to stop one hold-up problem typically creates another hold-up problem"...

"The cost of control is the loss of initiative"

⇒!!A downside of integration!!

- Weakness of (PR) theory: Reverse extreme as under Rent Seeking – Employees are now assumed to have no incentives to work, nor can they be motivated through managers using 'mechanisms' to do so.
    - Claim: Could adapt this to have integration yield only limited incentives without loss of generality

⇒ "Using a formal instrument to stop one hold-up problem typically creates another hold-up problem"...

"The cost of control is the loss of initiative"

⇒!!A downside of integration!!

- Weakness of (PR) theory: Reverse extreme as under Rent Seeking – Employees are now assumed to have no incentives to work, nor can they be motivated through managers using 'mechanisms' to do so.
  - Claim: Could adapt this to have integration yield only limited incentives without loss of generality
- Does this yield "crossing over"?

⇒ "Using a formal instrument to stop one hold-up problem typically creates another hold-up problem"...

"The cost of control is the loss of initiative"

⇒!!A downside of integration!!

- Weakness of (PR) theory: Reverse extreme as under Rent Seeking – Employees are now assumed to have no incentives to work, nor can they be motivated through managers using 'mechanisms' to do so.
  - Claim: Could adapt this to have integration yield only limited incentives without loss of generality
- Does this yield "crossing over"?
  - Perhaps if monitoring/incentives are more difficult for larger firms ... who "monitors the monitors"? Possible collusion.... (double markups in a sense?)

A Simple "complete" Model with noncontractible investment (From Goyal notes)

One downstream firm (D) and one upstream firm (U).

The upstream firm makes an input that is of value Q to the downstream firm. If they trade, D pays U a price (lowercase) p for this input.

A Simple "complete" Model with noncontractible investment (From Goyal notes)

One downstream firm (D) and one upstream firm (U).
The upstream firm makes an input that is of value Q to the downstream firm. If they trade, D pays U a price (lowercase) p for this input.

There is an alternative use of the inputs for, say, firm A, for which U can get a price (upper-case) P.
Second, firm U can make investments and these investments can increase the value of Q and P. In what follows we will assume that $Q > P$.

A Simple "complete" Model with noncontractible investment (From Goyal notes)

One downstream firm (D) and one upstream firm (U).
The upstream firm makes an input that is of value Q to the downstream firm. If they trade, D pays U a price (lowercase) p for this input.

There is an alternative use of the inputs for, say, firm A, for which U can get a price (upper-case) P.
Second, firm U can make investments and these investments can increase the value of Q and P. In what follows we will assume that $Q > P$.

This assumption means that it is efficient for U and D to agree on a price and that U should not sell to A.

A Simple "complete" Model with noncontractible investment (From Goyal notes)

One downstream firm (D) and one upstream firm (U).
The upstream firm makes an input that is of value Q to the downstream firm. If they trade, D pays U a price (lowercase) p for this input.

There is an alternative use of the inputs for, say, firm A, for which U can get a price (upper-case) P.
Second, firm U can make investments and these investments can increase the value of Q and P. In what follows we will assume that $Q > P$.

This assumption means that it is efficient for U and D to agree on a price and that U should not sell to A.

What are the terms of trade between U and D?
We assume that firms U and D can observe P and Q and so a simple way to proceed is to assume that the two have equal bargaining power (standard Nash bargaining). This means that they equally split the surplus: the price is then given by $p^* = (P + Q)/2$.

Note: the total bargaining surplus (gains from trade) is $Q - P$. Splitting this equally yields $\frac{Q-P}{2}$ bargaining surplus for each.

Note: the total bargaining surplus (gains from trade) is $Q - P$. Splitting this equally yields $\frac{Q-P}{2}$ bargaining surplus for each.

At price $p^* = (P + Q)/2$, D's bargaining surplus is $S_D = Q - (P + Q)/2 = \frac{Q-P.}{2}$.

Note: the total bargaining surplus (gains from trade) is $Q - P$.
Splitting this equally yields $\frac{Q-P}{2}$ bargaining surplus for each.

At price $p^* = (P+Q)/2$, D's bargaining surplus is $S_D = Q - (P+Q)/2 = \frac{Q-P.}{2}$.
At price $p^* = (P+Q)/2$, U's bargaining surplus is $S_U = \frac{(P+Q)}{2} - P = \frac{Q-P.}{2}$.

Thus, the bargaining surplus is split equally.
Remember – bargaining surplus is defined *relative to outside option*.

If bargaining occurs after investment choices, this may generate inefficient incentives for U:

If bargaining occurs after investment choices, this may generate inefficient incentives for U:

Assume that $Q = I_1 a_1 + \epsilon$, while $P = I_2 a_2 + \delta$, where ϵ and δ are, e.g., independently normally distributed with mean zero.

If bargaining occurs after investment choices, this may generate inefficient incentives for U:

Assume that $Q = I_1 a_1 + \epsilon$, while $P = I_2 a_2 + \delta$, where ϵ and δ are, e.g., independently normally distributed with mean zero.

We now examine incentives under integration and non-integration:

1. Non-Integration: If U sells to A it gets a price (upper-case) P. This possibility creates bargaining power for firm U and may induce investments that increase P even if this has no bearing on value of Q. To see this consider the case where Q is fixed in value ($I_1 = 0$) however P can be increased by investments $I_2 = 1$, i.e., $Q = \epsilon$, $P = a_2 + \delta$.

1. Non-Integration: If U sells to A it gets a price (upper-case) P. This possibility creates bargaining power for firm U and may induce investments that increase P even if this has no bearing on value of Q. To see this consider the case where Q is fixed in value ($I_1 = 0$) however P can be increased by investments $I_2 = 1$, i.e., $Q = \epsilon$, $P = a_2 + \delta$.

U wants to maximize the price he receives minus his costs, i.e., $\frac{P+Q}{2} - c(a_1, a_2)$ Simple calculations then show that U will choose a^* such that $1/2 = c'(a^*)$, where $c(.)$ is an increasing and convex cost function. This investment is simply a waste of resources since the surplus generated is $Q - c(a^*)$.

1. Non-Integration: If U sells to A it gets a price (upper-case) P. This possibility creates bargaining power for firm U and may induce investments that increase P even if this has no bearing on value of Q. To see this consider the case where Q is fixed in value ($I_1 = 0$) however P can be increased by investments $I_2 = 1$, i.e., $Q = \epsilon$, $P = a_2 + \delta$.
U wants to maximize the price he receives minus his costs, i.e., $\frac{P+Q}{2} - c(a_1, a_2)$ Simple calculations then show that U will choose a^* such that $1/2 = c'(a^*)$, where $c(.)$ is an increasing and convex cost function. This investment is simply a waste of resources since the surplus generated is $Q - c(a^*)$.

It follows that integration will resolve this problem: a take-over of U by D will mean that U will have no power to sell to A and so will have no incentive to make investments in P, which are wasteful. In this setting, integration prevents U "holding up" D. If integrates, $P^* = 0, a_1 = a_2 = 0$

Note: Even under non-integration, U and D could agree on a price in advance and avoid this problem; but in an unanticipated contingency we are back to the case above.

Marriage analogy:
a_1 is like learning how to prepare nice meals for one's spouse, or learning about their interests
$a_2$is like making oneself appealing to other suitors

2. Integration: Assume that integration involves D buying all rights to the equipment and hence output of U. In particular, once integration takes place U has no rights to sell to A.

2. Integration: Assume that integration involves D buying all rights to the equipment and hence output of U. In particular, once integration takes place U has no rights to sell to A.

Now suppose that $Q = a_1 + \epsilon$ and P is fixed in value. Upon take-over U has no rights to surplus and so U has no incentives to make any investments at all (again, relevant for uncontractable contingencies).

2. Integration: Assume that integration involves D buying all rights to the equipment and hence output of U. In particular, once integration takes place U has no rights to sell to A.

Now suppose that $Q = a_1 + \epsilon$ and P is fixed in value. Upon take-over U has no rights to surplus and so U has no incentives to make any investments at all (again, relevant for uncontractable contingencies).

...'Since we got married, you don't even make an effort!'

2. Integration: Assume that integration involves D buying all rights to the equipment and hence output of U. In particular, once integration takes place U has no rights to sell to A.

Now suppose that $Q = a_1 + \epsilon$ and P is fixed in value. Upon take-over U has no rights to surplus and so U has no incentives to make any investments at all (again, relevant for uncontractable contingencies).

...'Since we got married, you don't even make an effort!'

However, non-integration provides incentives to firm U, since it then gets a price $\frac{1}{2}(Q + P)$ (in the contingency), and so will choose an action $\bar{a}_1$ such that $1/2 = c'(\bar{a}_1)$.

2. Integration: Assume that integration involves D buying all rights to the equipment and hence output of U. In particular, once integration takes place U has no rights to sell to A.

Now suppose that $Q = a_1 + \epsilon$ and P is fixed in value. Upon take-over U has no rights to surplus and so U has no incentives to make any investments at all (again, relevant for uncontractable contingencies).

...'Since we got married, you don't even make an effort!'

However, non-integration provides incentives to firm U, since it then gets a price $\frac{1}{2}(Q + P)$ (in the contingency), and so will choose an action $\bar{a}_1$ such that $1/2 = c'(\bar{a}_1)$.
In such a case, if $Q(\bar{a}_1) - c(\bar{a}_1) > Q$, i.e., if the investment is technically efficient, then non-integration is optimal.

1. Hold up creates incentives for parties

1. Hold up creates incentives for parties
2. Incentives may be good, bad, or both

1. Hold up creates incentives for parties
2. Incentives may be good, bad, or both
3. Ownership can stop hold up

1. Hold up creates incentives for parties
2. Incentives may be good, bad, or both
3. Ownership can stop hold up
4. Solving one hold up problem can potentially create another hold up problem

1. Hold up creates incentives for parties
2. Incentives may be good, bad, or both
3. Ownership can stop hold up
4. Solving one hold up problem can potentially create another hold up problem
5. Integration may thus be worse than non-integration in some situations.

Elemental Property Rights Theory of the Firm (Grossman and Hart; Gibbons version)

Timing

1. Parties negotiate decision right (ownership) (cannot contract over d)

Elemental Property Rights Theory of the Firm (Grossman and Hart; Gibbons version)

Timing

1. Parties negotiate decision right (ownership) (cannot contract over d)
2. Parties $i = 1, 2$ choose $a_i \in A_i$ ... *the potentially distorted 'investment'* . (i.e., party 1 chooses a_1, party 2 chooses a_2)

Elemental Property Rights Theory of the Firm (Grossman and Hart; Gibbons version)

Timing

1. Parties negotiate decision right (ownership) (cannot contract over d)
2. Parties $i = 1, 2$ choose $a_i \in A_i$ ... *the potentially distorted 'investment'* . (i.e., party 1 chooses a_1, party 2 chooses a_2)
3. Both parties observe a_1, a_2, state $s \in S$ ... *the uncontracted contingency*

Elemental Property Rights Theory of the Firm (Grossman and Hart; Gibbons version)

Timing

1. Parties negotiate decision right (ownership) (cannot contract over d)
2. Parties $i = 1, 2$ choose $a_i \in A_i$ ... *the potentially distorted 'investment'* . (i.e., party 1 chooses a_1, party 2 chooses a_2)
3. Both parties observe a_1, a_2, state $s \in S$ ... *the uncontracted contingency*
4. Negotiate/choose $d \in D$ (decision) ...*potential for hold-up* (Note efficient bargaining ex-post over choice of d)

Elemental Property Rights Theory of the Firm (Grossman and Hart; Gibbons version)

Timing

1. Parties negotiate decision right (ownership) (cannot contract over d)
2. Parties $i = 1, 2$ choose $a_i \in A_i$ ... *the potentially distorted 'investment'* . (i.e., party 1 chooses a_1, party 2 chooses a_2)
3. Both parties observe a_1, a_2, state $s \in S$ ... *the uncontracted contingency*
4. Negotiate/choose $d \in D$ (decision) ...*potential for hold-up* (Note efficient bargaining ex-post over choice of d)
5. Payoffs: $V_i \equiv U_i(a_1, a_2, s, d) + \textit{transfers} - c_i(a_i)$

Features

- a_i's and U_i's are 'non-contractible' (unobserved or unverifiable)

Features

- a_i's and U_i's are 'non-contractible' (unobserved or unverifiable)
- d contractible in stage 4 $\Longrightarrow$ 'ex post efficient'

Features

- a_i's and U_i's are 'non-contractible' (unobserved or unverifiable)
- d contractible in stage 4 $\Longrightarrow$ 'ex post efficient'

Features

- a_i's and U_i's are 'non-contractible' (unobserved or unverifiable)
- d contractible in stage 4 $\Longrightarrow$ 'ex post efficient'

Argument:

Control in stage 1 $\Longrightarrow$ Allocation of surplus in stage 4 $\Longrightarrow$ Investment in stage 2 $\Longrightarrow$ Surplus/Payoffs at end

Backwards induction

Stage 4

Efficient decision:

$$
\begin{aligned}
d^*(a_1, a_2, s) &= \arg\max_{d \in D} [U_1(a_1, a_2, s, d) + U_2(a_1, a_2, s, d)] \\
&\implies U_i^*(a_1, a_2, s) \equiv U_i^*(a_1, a_2, s, d^*(a_1, a_2, s))
\end{aligned}
$$

Backwards induction

Stage 4

Efficient decision:

$$\begin{aligned} d^*(a_1, a_2, s) &= \arg\max_{d \in D} [U_1(a_1, a_2, s, d) + U_2(a_1, a_2, s, d)] \\ &\implies U_i^*(a_1, a_2, s) \equiv U_i^*(a_1, a_2, s, d^*(a_1, a_2, s)) \end{aligned}$$

i's (owner's) preferred decision (but-for the payments):

$$\begin{aligned} d_i(a_1, a_2, s) &= \arg\max_{d \in D} U_i(a_1, a_2, s, d) \\ &\implies U_j^i \equiv U_j(a_1, a_2, s, d_i(a_1, a_2, s)) \end{aligned}$$

Note (potential) realized utility of j depends on i's decision (where j may or may not equal i), chosen to maximize *i's* utility.

Stage 4
Efficient decision:

$$\begin{aligned} d^*(a_1, a_2, s) &= \arg\max_{d \in D} [U_1(a_1, a_2, s, d) + U_2(a_1, a_2, s, d)] \\ &\implies U_i^*(a_1, a_2, s) \equiv U_i^*(a_1, a_2, s, d^*(a_1, a_2, s)) \end{aligned}$$

i's (owner's) preferred decision (but-for the payments):

$$\begin{aligned} d_i(a_1, a_2, s) &= \arg\max_{d \in D} U_i(a_1, a_2, s, d) \\ &\implies U_j^i \equiv U_j(a_1, a_2, s, d_i(a_1, a_2, s)) \end{aligned}$$

Note (potential) realized utility of j depends on i's decision (where j may or may not equal i), chosen to maximize *i's* utility.
Preference asymmetry (of a certain sort) $\implies$

$$U_1^*(a_1, a_2, s) + U_2^*(a_1, a_2, s) > U_1^i(a_1, a_2, s) + U_2^i(a_1, a_2, s)$$

for $i = 1$ or $i = 2$

Stage 4

Efficient decision:

$$d^*(a_1, a_2, s) = \arg\max_{d \in D} [U_1(a_1, a_2, s, d) + U_2(a_1, a_2, s, d)]$$
$$\implies U_i^*(a_1, a_2, s) \equiv U_i^*(a_1, a_2, s, d^*(a_1, a_2, s))$$

i's (owner's) preferred decision (but-for the payments):

$$d_i(a_1, a_2, s) = \arg\max_{d \in D} U_i(a_1, a_2, s, d)$$
$$\implies U_j^i \equiv U_j(a_1, a_2, s, d_i(a_1, a_2, s))$$

Note (potential) realized utility of j depends on i's decision (where j may or may not equal i), chosen to maximize *i's* utility.

Preference asymmetry (of a certain sort) $\implies$

$$U_1^*(a_1, a_2, s) + U_2^*(a_1, a_2, s) > U_1^i(a_1, a_2, s) + U_2^i(a_1, a_2, s)$$

for $i = 1$ or $i = 2$

$\implies$ incentive to negotiate to attain d^*

Stage 4 'Nash' Bargaining

Nash bargaining basically maximizes the bargaining surplus and splits it evenly, or according to the bargaining weights.

'Threat points' (when i has ownership):

$$U_1^i(a_1, a_2, s), U_2^i(a_1, a_2, s)$$

Stage 4 'Nash' Bargaining

Nash bargaining basically maximizes the bargaining surplus and splits it evenly, or according to the bargaining weights.

'Threat points' (when i has ownership):

$$U_1^i(a_1, a_2, s), U_2^i(a_1, a_2, s)$$

Nash bargaining axioms (with equal bargaining weights)$\Longrightarrow$ d and p maximize product of 'surpluses relative to threat points,' i.e., solve:

$$\max_{d \in D, p} \left[U_1(a_1, a_2, s, d) + p - U_1^i(a_1, a_2, s)\right]$$
$$\times \left[U_2(a_1, a_2, s, d) - p - U_2^i(a_1, a_2, s)\right]$$

where p is a (positive or negative) payment from 2 to 1 for which i agrees to choose

Setting p to maximize this $\Longrightarrow$

$$p = \frac{1}{2}\left(\begin{array}{c}[U_2(a_1, a_2, s, d) - U_2^i(a_1, a_2, s)] \\ -[U_1(a_1, a_2, s, d) - U_1^i(a_1, a_2, s)]\end{array}\right)$$

Setting p to maximize this $\Longrightarrow$

$$
\begin{aligned}
p &= \frac{1}{2}\left(\begin{array}{c} [U_2(a_1,a_2,s,d) - U_2^i(a_1,a_2,s)] \\ -[U_1(a_1,a_2,s,d) - U_1^i(a_1,a_2,s)] \end{array}\right) \\
&\Longrightarrow \text{(substituting into the previous expression)} \\
d &= \arg\max_{d\in D} \frac{1}{4}[U_1(a_1,a_2,s,d) + U_2(a_1,a_2,s,d) \\
&\quad -U_1^i(a_1,a_2,s) - U_2^i(a_1,a_2,s)]^2
\end{aligned}
$$

Setting p to maximize this $\Longrightarrow$

$$\begin{aligned} p &= \frac{1}{2}\left(\begin{array}{c} [U_2(a_1, a_2, s, d) - U_2^i(a_1, a_2, s)] \\ -[U_1(a_1, a_2, s, d) - U_1^i(a_1, a_2, s)] \end{array} \right) \\ &\Longrightarrow \text{(substituting into the previous expression)} \\ d &= \arg\max_{d \in D} \frac{1}{4}[U_1(a_1, a_2, s, d) + U_2(a_1, a_2, s, d) \\ &\quad -U_1^i(a_1, a_2, s) - U_2^i(a_1, a_2, s)]^2 \end{aligned}$$

Since the latter terms are unaffected by d and the first 2 terms represent the surplus we have an efficient decision (note to maximize a squared term we must maximize its contents' absolute value):

$$d = d^*(a_1, a_2, s)$$

Noting the payment from 2 to 1 when i has control is

$$p^* = \frac{1}{2}\left(\begin{array}{c} [U_2^*(a_1, a_2, s) - U_2^i(a_1, a_2, s)] \\ -[U_1^*(a_1, a_2, s) - U_1^i(a_1, a_2, s)] \end{array}\right)$$

Noting the payment from 2 to 1 when i has control is

$$p^* = \frac{1}{2}\left(\begin{array}{c} \left[U_2^*(a_1, a_2, s) - U_2^i(a_1, a_2, s)\right] \\ -\left[U_1^*(a_1, a_2, s) - U_1^i(a_1, a_2, s)\right] \end{array}\right)$$

Whether I am 2 or 1, through p^*, I lose $\frac{1}{2}$ of my own (conditional on actions) optimal utility, but gain $\frac{1}{2}$ of other guys' (conditional) optimal utility; I also gain $\frac{1}{2}$ of my own threat point, but lose $\frac{1}{2}$ of other guy's threat point. These threat points (hence payoffs), depend on ownership ($i = 1$ or $i = 2$).

Noting the payment from 2 to 1 when i has control is

$$p^* = \frac{1}{2}\begin{pmatrix} [U_2^*(a_1, a_2, s) - U_2^i(a_1, a_2, s)] \\ -[U_1^*(a_1, a_2, s) - U_1^i(a_1, a_2, s)] \end{pmatrix}$$

Whether I am 2 or 1, through p^*, I lose $\frac{1}{2}$ of my own (conditional on actions) optimal utility, but gain $\frac{1}{2}$ of other guys' (conditional) optimal utility; I also gain $\frac{1}{2}$ of my own threat point, but lose $\frac{1}{2}$ of other guy's threat point. These threat points (hence payoffs), depend on ownership ($i = 1$ or $i = 2$). Summing utilities from outcomes, payments, and actions, j's net payoff if i is owner ($i = j$ or $i \neq j$; but $j \neq k$; j may be owner or not, but k is the "other guy") are:

Noting the payment from 2 to 1 when i has control is

$$p^* = \frac{1}{2}\left(\begin{array}{c} \left[U_2^*(a_1,a_2,s) - U_2^i(a_1,a_2,s)\right] \\ -\left[U_1^*(a_1,a_2,s) - U_1^i(a_1,a_2,s)\right] \end{array} \right)$$

Whether I am 2 or 1, through p^*, I lose $\frac{1}{2}$ of my own (conditional on actions) optimal utility, but gain $\frac{1}{2}$ of other guys' (conditional) optimal utility; I also gain $\frac{1}{2}$ of my own threat point, but lose $\frac{1}{2}$ of other guy's threat point. These threat points (hence payoffs), depend on ownership ($i = 1$ or $i = 2$). Summing utilities from outcomes, payments, and actions, j's net payoff if i is owner ($i = j$ or $i \neq j$; but $j \neq k$; j may be owner or not, but k is the "other guy") are:

$$\begin{aligned} NP_j^i(a_1,a_2,s) &\equiv \frac{1}{2}\left[U_j^*(a_1,a_2,s) + U_k^*(a_1,a_2,s)\right] \\ &\quad + \frac{1}{2}\left[U_j^i(a_1,a_2,s) - U_k^i(a_1,a_2,s)\right] - c_j(a_j) \\ &\equiv \frac{1}{2}ETS(a_1,a_2,s) + \frac{1}{2}TPD_j^i(a_1,a_2,s) - c_j(a_j) \end{aligned}$$

Where ETS is the "Efficient Total Surplus" and TPD_j^i is the "Threat Point Difference" (j's threat point minus k's).

$$NP_j^i(a_1, a_2, s) \equiv \frac{1}{2}ETS(a_1, a_2, s) + \frac{1}{2}TPD_j^i(a_1, a_2, s) - c_j(a_j)$$

So, in choosing a_j, each player j weighs the surplus (ETS) and the bargaining position (TPD_j^i) equally.
I.e., a player will act partly to 'manipulate the threat point.'

Gibbons: "we would like to find a governance structure such that the existing half strength incentives from [TPD] closely approximate the missing half-strength incentives from [ETS]"

Solving for (stage 2) actions:

Under player i's ownership, each player j ($i = j$ or $i \neq j$) chooses a_j so that

$$a_j = \arg\max_{a_j \in A_j} E_s \left[NP_j^i(a_i, a_j, s) \right]$$

$$\implies \quad \text{Nash equilibrium actions } \left[a_1^{*(i)}, a_2^{*(i)} \right]$$

Note: E_s represents the expected value, integrated over the random variable s.

Solving for (stage 2) actions:

Under player i's ownership, each player j ($i = j$ or $i \neq j$) chooses a_j so that

$$a_j = \arg\max_{a_j \in A_j} E_s \left[NP_j^i(a_i, a_j, s) \right]$$

$$\Longrightarrow \quad \text{Nash equilibrium actions } \left[a_1^{*(i)}, a_2^{*(i)} \right]$$

Note: E_s represents the expected value, integrated over the random variable s.
Note: $a_1^{*(i)}, a_2^{*(i)}$ are not (necessarily) the efficient actions – just the Nash equilibrium actions under i's ownership

Solving for (stage 2) actions:

Under player i's ownership, each player j ($i = j$ or $i \neq j$) chooses a_j so that

$$a_j = \arg\max_{a_j \in A_j} E_s \left[NP_j^i(a_i, a_j, s) \right]$$

$$\Longrightarrow \quad \text{Nash equilibrium actions } \left[a_1^{*(i)}, a_2^{*(i)} \right]$$

Note: E_s represents the expected value, integrated over the random variable s.
Note: $a_1^{*(i)}, a_2^{*(i)}$ are not (necessarily) the efficient actions – just the Nash equilibrium actions under i's ownership

$$\Longrightarrow TS^i = E_s \left[U_1^*(a_1^{*(i)}, a_2^{*(i)}, s) + U_2^*(a_1^{*(i)}, a_2^{*(i)}, s) \right] - c_1(a_1^{*(i)}) - c_2(a_2^{*(i)})$$

Solving for (stage 2) actions:

Under player i's ownership, each player j ($i = j$ or $i \neq j$) chooses a_j so that

$$a_j = \arg\max_{a_j \in A_j} E_s \left[NP_j^i(a_i, a_j, s) \right]$$

$$\Longrightarrow \quad \text{Nash equilibrium actions } \left[a_1^{*(i)}, a_2^{*(i)} \right]$$

Note: E_s represents the expected value, integrated over the random variable s.
Note: $a_1^{*(i)}, a_2^{*(i)}$ are not (necessarily) the efficient actions – just the Nash equilibrium actions under i's ownership

$$\Longrightarrow TS^i = E_s \left[U_1^*(a_1^{*(i)}, a_2^{*(i)}, s) + U_2^*(a_1^{*(i)}, a_2^{*(i)}, s) \right] - c_1(a_1^{*(i)}) - c_2(a_2^{*(i)})$$

Stage 1: Efficient negotiation will set ownership to maximize TS^i, comparing TS^1 and TS^2

Generally (speculation here), will want to give ownership to the party:

1. ?? Whose level of investment/action (a_i) is most crucial to maximizing the joint surplus, thus most needs to be motivated (via the surplus she will extract in stage 4 from ownership) to act optimally. ...

Generally (speculation here), will want to give ownership to the party:

1. ?? Whose level of investment/action (a_i) is most crucial to maximizing the joint surplus, thus most needs to be motivated (via the surplus she will extract in stage 4 from ownership) to act optimally. ...
2. ?? ? Whose (selfishly) optimal decision ($d_i(\cdot)$) is the least harmful to the other party; thus will have less room for 'holding up' the other party, likely minimising both sides' incentives to take actions solely to improve their threat points.

Generally (speculation here), will want to give ownership to the party:

1. ?? Whose level of investment/action (a_i) is most crucial to maximizing the joint surplus, thus most needs to be motivated (via the surplus she will extract in stage 4 from ownership) to act optimally. ...
2. ?? ? Whose (selfishly) optimal decision ($d_i(\cdot)$) is the least harmful to the other party; thus will have less room for 'holding up' the other party, likely minimising both sides' incentives to take actions solely to improve their threat points.
3. ?? Under whose (selfishly) optimal decision, 'wasteful' investments (extending this to a multiple -action case) have little effect on threat points. This may require that ($d_i(\cdot)$) is itself little affected by the investments.

Generally (speculation here), will want to give ownership to the party:

1. ?? Whose level of investment/action (a_i) is most crucial to maximizing the joint surplus, thus most needs to be motivated (via the surplus she will extract in stage 4 from ownership) to act optimally. ...
2. ?? ? Whose (selfishly) optimal decision ($d_i(\cdot)$) is the least harmful to the other party; thus will have less room for 'holding up' the other party, likely minimising both sides' incentives to take actions solely to improve their threat points.
3. ?? Under whose (selfishly) optimal decision, 'wasteful' investments (extending this to a multiple -action case) have little effect on threat points. This may require that ($d_i(\cdot)$) is itself little affected by the investments.
4. **Key:** Under whose (selfishly) optimal decision investments have similar effects on threat points as their effects under the optimal decision. Hence if this party has ownership, each party, in trying to improve their threat point, will also tend to increase the total surplus.

Generally (speculation here), will want to give ownership to the party:

1. ?? Whose level of investment/action (a_i) is most crucial to maximizing the joint surplus, thus most needs to be motivated (via the surplus she will extract in stage 4 from ownership) to act optimally. ...
2. ?? ? Whose (selfishly) optimal decision ($d_i(\cdot)$) is the least harmful to the other party; thus will have less room for 'holding up' the other party, likely minimising both sides' incentives to take actions solely to improve their threat points.
3. ?? Under whose (selfishly) optimal decision, 'wasteful' investments (extending this to a multiple -action case) have little effect on threat points. This may require that ($d_i(\cdot)$) is itself little affected by the investments.
4. **Key:** Under whose (selfishly) optimal decision investments have similar effects on threat points as their effects under the optimal decision. Hence if this party has ownership, each party, in trying to improve their threat point, will also tend to increase the total surplus.

Generally (speculation here), will want to give ownership to the party:

1. ?? Whose level of investment/action (a_i) is most crucial to maximizing the joint surplus, thus most needs to be motivated (via the surplus she will extract in stage 4 from ownership) to act optimally. ...
2. ?? ? Whose (selfishly) optimal decision ($d_i(\cdot)$) is the least harmful to the other party; thus will have less room for 'holding up' the other party, likely minimising both sides' incentives to take actions solely to improve their threat points.
3. ?? Under whose (selfishly) optimal decision, 'wasteful' investments (extending this to a multiple -action case) have little effect on threat points. This may require that ($d_i(\cdot)$) is itself little affected by the investments.
4. **Key:** Under whose (selfishly) optimal decision investments have similar effects on threat points as their effects under the optimal decision. Hence if this party has ownership, each party, in trying to improve their threat point, will also tend to increase the total surplus.

Example of unverifiable situation: The Singing Frog