EC992-G-AU **UNIVERSITY OF ESSEX**

GRADUATE EXAMINATIONS 2010

ADVANCED MICROECONOMICS

Time allowed: 2 hours.

This paper contains FOUR questions, divided in two sections, A and B.
Section A contains TWO questions. Section B contains TWO questions.
Answer ONE question from section A and ONE question from section B.
Each question in section A carries 50 weight.
Each question in section B carries 50 weight.
Candidates are allowed to bring into the examination room: calculators (hand held, containing no textual information).

Section A: Answer ONE question.

1. Consider the following labor market model. There are many identical firms that can hire workers. Each produces the same output using an identical constant returns to scale technology in which labor is the only input. The firms are risk neutral, seek to maximise their expected profits, and act as price takers. Workers differ in the number of units of output they produce if hired by a firm, which is denoted by θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in \{\underline{\theta}, \bar{\theta}\}$. A worker can decide whether to work or stay home. The utility to stay home of a worker θ is $r(\theta)$, where $r(\theta)$ is continuous and strictly decreasing function of θ.

(a) [5 marks] Define the notion of competitive equilibrium in this competitive labor market model with unobservable worker productivity.

(b) [10 marks] Show that the more capable workers are the ones choosing to work at any given wage.

(c) [15 marks] Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto Efficient.

(d) [20 marks] Suppose that there exists a θ^* such that $r(\theta) < \theta^*$ for $\theta > \theta^*$ and $r(\theta) > \theta^*$ for $\theta < \theta^*$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto optimal allocation of workers.

2. A worker knows her talent $\theta \in \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L > 0$. The employer does not know the talent of the worker but believes that $\lambda = \Pr(\theta = \theta_H) \in (0, 1)$. In the first stage the worker decides a level of education $e > 0$. The employer observes the level of education chosen by the worker and offers the worker a wage which equals the worker's expected productivity. Finally, the worker accept or reject the offer.

If a worker of type θ receives education e, his productivity is $\theta(1+e)$. The cost of obtaining education level e for a type θ worker is given by the twice continuously differentiable function $c(e, \theta)$, with $c(0, \theta) = 0$, $c_e(e, \theta) > 0$, $c_{ee}(0, \theta) > 0$, $c_\theta(e, \theta) < 0$ for all $e > 0$ and $c_{e\theta}(e, \theta) < 0$, where subscripts denote partial derivatives. The utility of a type θ worker who chooses education level e, receives wage w and accepts the offer is $u(w, e|\theta) = w - c(e, \theta)$. It is also assumed that a worker of talent θ can earn $r(\theta)$ by working home and that $r(\theta_L) = r(\theta_H) = 0$.

(a) [15 marks] Identify the pooling perfect bayesian equilibrium and illustrate it graphically.

(b) [15 marks] Identify the separating bayesian equilibria and illustrate them graphically.

(c) Suppose now there is perfect information.

(c_1) [10 marks] Find the competitive equilibrium.

(c_2) [10 marks] Relate the separating and pooling bayesian equilibria to the perfect information competitive outcome.

End of Section A

Section B: Answer ONE question.

3. A firm employs an agent who is risk-neutral, but has limited liability, i.e., they cannot be paid a negative wage. There is no individual rationality constraint. The agent can choose action $a \in \{L, H\}$ and the cost of action a is $c(a)$. It is assumed that $c(L) = 0$ and $c(H) = c > 0$. There are two possible outputs $\{q_L, q_H\}$. The high output occurs with probability p_L or p_H if the agent takes action L or H, respectively. It is assumed that $0 < p_L < p_H < 1$. If the agent receives a wage of w and chooses action a, his payoff is $w - c(a)$. If the realised output is q and the principal pays wage w, his payoff is $q - w$.

(a) [10 marks] Characterise the optimal wages and action.

(b) Suppose now that there are two types of agents $i \in \{1, 2\}$. The principal cannot observe an agent's type but believes the probability of either type is $1/2$. The agents are identical except for their cost of taking the action: for agent $i \in \{1, 2\}$ the cost of $a \in \{L, H\}$ is: $c_i(L) = 0$ and $c_i(H) = c_i$, with $c_2 > c_1$.

(b_1) [10 marks] Show that it is impossible for the principal to implement $\{a_1, a_2\} = \{L, H\}$.

(b_2) [10 marks] What are the optimal wages if the principal wishes to implement $\{a_1, a_2\} = \{H, H\}$?

(b_3) [20 marks] What are the optimal wages if the principal wishes to implement $\{a_1, a_2\} = \{H, L\}$?

4. A principal employs an agent to work on a project. The worker chooses unobserved effort $e \in \{L, H\}$ at costs c_L and c_H, with $0 < c_L < c_H$. The project succeeds with probability p_H if $e = H$ and p_L if $e = L$, where $1 > p_H > p_L > 0$. The principal pays w_0 if the project fails and w_1 if it succeeds. The agent's utility is given by $u(w_s) - c_e$, where $s \in \{0, 1\}$. Utility $u(\cdot)$ is strictly increasing and strictly concave. The agent has reservation utility $\underline{U}$. The principal's profit is $x_s - w_s$, where x_1 is the output when the project succeeds, and x_0 is the output when the project fails, and $x_1 > x_0$.

(a) Suppose the principal chooses to implement $e = L$.

- (a_1) [5 marks] Write down the principal's maximisation problem.
- (a_2) [10 marks] What wages will the principal offer in the profit-maximising contract?

(b) [20 marks] Now suppose the principal chooses to implement $e = H$.

- (b_1) [10 marks] Write down the principal's maximisation problem.
- (b_2) [15 marks] What wages will the principal offer in the profit-maximising contract?
- (b_3) [10 marks] Show that in the profit-maximising contract derived in part b_2 it holds that $w_1 > w_0$.

End of Section B

End of Paper