EC992-G-AU **UNIVERSITY OF ESSEX**

GRADUATE EXAMINATIONS 2010

ADVANCED MICROECONOMICS

Time allowed: 2 hours.

This paper contains FOUR questions, divided in two sections, A and B.
Section A contains TWO questions. Section B contains TWO questions.
Answer ONE question from section A and ONE question from section B.
Each question in section A carries 50 weight.
Each question in section B carries 50 weight.
Candidates are allowed to bring into the examination room: calculators (hand held, containing no textual information).

Section A: Answer ONE question.

1. Consider the following labor market model. There are many identical firms that can hire workers. Each produces the same output using an identical constant returns to scale technology in which labor is the only input. The firms are risk neutral, seek to maximise their expected profits, and act as price takers. Workers differ in the number of units of output they produce if hired by a firm, denoted by θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in \{\underline{\theta}, \bar{\theta}\}$. A worker can decide whether to work or stay home. The utility to stay home of a worker θ is $r(\theta)$, where $r(\theta)$ is continuous and strictly decreasing function of θ.

(a) [5 marks] Define the notion of competitive equilibrium in this competitive labor market model with unobservable worker productivity.

(b) [10 marks] Show that the more capable workers are the ones choosing to work at any given wage.

(c) [15 marks] Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto Efficient.

(d) [20 marks] Suppose that there exists a θ^* such that $r(\theta) < \theta^*$ for $\theta > \theta^*$ and $r(\theta) > \theta^*$ for $\theta < \theta^*$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto optimal allocation of workers.

SOLUTION 1.

a A competitive equilibrium is a wage w and a set of employed workers Σ such that: $w = E[\theta|\Sigma]$ and Σ is the set of all workers such that $r(\theta) < w$.

b Suppose firms offer a wage w. All workers of type θ, with $r(\theta) \leq w$ will accept the wage and work. Suppose there exists a θ^* with $r(\theta^*) = w$. Then all worker of type $\theta \geq \theta^*$ will work since $r(\theta) \leq r(\theta^*) = w$ and r is decreasing. Thus, the more capable workers are the ones who will work at any given work.

c Firms can offer a wage $w = \bar{\theta}$ and since $r(\bar{\theta}) > \bar{\theta}$ no worker of type $\bar{\theta}$ will work. The previous part then implies that no worker of any type will work. therefore, the competitive equilibrium is Pareto efficient, i.e., nobody will work.

d If $w = \theta^*$ only workers with $\theta \geq \theta^*$ will accept to work at w. But then the expected producity is higher than θ^* and therefore higher than the wage offered: firms demand more workes than there are in supply and the market will not clear. If $w < \theta^*$ only workers between θ_l and θ^* work, where $r(\theta_l) = w$ (because r is a decreasing function) But then the expected productivity is higher than θ_l and so firms demand more workers than there are in supply and market does not clear. Thus, in a competitive equilibrium $w > \theta^*$, which implies that some worker with $\theta < \theta^*$ will accept the job, and so there is over employment. In fact, in an equilibrium with perfect information only workers of type higher than θ^* will work.

2. A worker knows her talent $\theta \in \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L > 0$. The employer does not know the talent of the worker but believes that $\lambda = \Pr(\theta = \theta_H) \in (0,1)$. In the first stage the worker decides a level of education $e > 0$. The employer observes the level of education chosen by the worker and offers the worker a wage which equals the worker's expected productivity. Finally, the worker accept or reject the offer.

If a worker of type θ receives education e, his productivity is $\theta(1+e)$. The cost of obtaining education level e for a type θ worker is given by the twice continuously differentiable function $c(e,\theta)$, with $c(0,\theta) = 0$, $c_e(e,\theta) > 0$, $c_{ee}(0,\theta) > 0$, $c_\theta(e,\theta) < 0$ for all $e > 0$ and $c_{e\theta}(e,\theta) < 0$, where subscripts denote partial derivatives. The utility of a type θ worker who chooses education level e, receives wage w and accepts the offer is $u(w, e|\theta) = w - c(e,\theta)$. It is also assumed that a worker of θ can earn $r(\theta)$ by working home and that $r(\theta_L) = r(\theta_H) = 0$.

(a) [15 marks] Identify the pooling perfect bayesian equilibrium and illustrate it graphically.

(b) [15 marks] Identify the separating bayesian equilibria and illustrate them graphically.

(c) Suppose now there is perfect information.

   (c_1) [10 marks] Find the competitive equilibrium.

   (c_2) [10 marks] Relate the separating and pooling bayesian equilibria to the perfect information competitive outcome.

SOLUTION 2

Part a. The pooling equilibrium of this model is as follows: $[e^*, (1-\lambda)(\theta_L + \theta_L e^*) + \lambda(\theta_H + \theta_H e^*)]$, with $e^* \in [0, e']$. The equilibrium is depicted in Figure 13.C.2(d).

Part b. The separating equilibrium of this model is as follows: $[e_L^*, \theta_L + \theta_L e_L^*]$ and $[e_H^*, \theta_H + \theta_H e_H^*]$, where e_L^* solves $c_e(e_L^*, \theta_L) = \theta_L$, and $e_H^* \in [e', e'']$. The equilibrium is depiceted in Figure13.C.2(c).

Part c1. At the competitive equilibrium with perfect information, type θ_x gets education e_x^*, such that $c_e(e_x^*, \theta_x) = \theta_x$, $x = H, L$. The wage for θ_H is $w_H^* = \theta_H + \theta_H e_H^*$ and $w_L^* = \theta_L + \theta_H e_L^*$. Note that the competitive outcome is Pareto efficient.

Part c2. Type θ_L in the separating equilibrium behaves as under complete information. Generally the level of effort of the high type is inefficient. Yet, note that as education affects productivity, it is possible to find situation in which the high productivity type may also obtain the optimal level of education. A good student would provide a graphical illustration of two cases: one in which optimal effort is obtained by the high type and the other when optimal effort is not obtained.

End of Section A

Section B: Answer ONE question.

3. A firm employs an agent who is risk-neutral, but has limited liability, i.e., they cannot be paid a negative wage. There is no individual rationality constraint. The agent can choose action $a \in \{L, H\}$ and the cost of action a is $c(a)$. It is assumed that $c(L) = 0$ and $c(H) = c > 0$. There are two possible outputs $\{q_L, q_H\}$. The high output occurs with probability p_L or p_H if the agent takes action L or H, respectively. It is assumed that $0 < p_L < p_H < 1$. If the agent receives a wage of w and chooses action a, his payoff is $w - c(a)$. If the realised output is q and the principal pays wage w, his payoff is $q - w$.

(a) [10 marks] Characterise the optimal wages and action.

(b) Suppose now that there are two types of agents $i \in \{, 2$. The principal cannot observe an agent's type but believes the probability of either type is $1/2$. The agents are identical except for their cost of taking the action: for agent $i\{, 2\}$ the cost of $a \in \{L, H\}$ is: $c_i(L) = 0$ and $c_i(H) = c_i$, with $c_2 > c_1$.

(b_1) [10 marks] Show that it is impossible for the principal to implement $\{a_1, a_2\} = \{L, H\}$.

(b_2) [10 marks] What are the optimal wages if the principal wishes to implement $\{a_1, a_2\} = \{H, H\}$?

(b_3) [20 marks] What are the optimal wages if the principal wishes to implement $\{a_1, a_2\} = \{H, L\}$?

SOLUTION 3.

a. To implement to low action set $w_H = w_L = 0$. The profit is $q_L + p_H\Delta(q)$, where $\Delta(q) = q_H - q_L$. The implement the high action set $w_L = 0$ and $w_H = c/(\Delta(p))$, where $\Delta(p) = p_H - p_L$. Profit is $q_L + p_H[\Delta(q) - c/\Delta(p)]$.

b_1 This is impossible to implement because if the high-cost agent is wiling to exert effort, the so is the low-cost agent.

b_2To implement $\{H, H\}$ set $\{w_L^i, w_H^i\} = \{0, c_2/\Delta(p)\}$, for $i = 1, 2$. This satisfies all the IC constraints. First, no agent whishes to pretend to be the other. Second, both types are willing to exert effort. Profit is then $q_L + p_H[\Delta(q) - c_2/\Delta(p)]$

b_3 Consider $\{H, L\}$. The IC constraints for type 1 are

$$w_L^1 + p_H(w_H^1 - w_L^1) - c_1 \geq w_L^1 + p_L(w_H^1 - w_L^1)$$
$$w_L^1 + p_H(w_H^1 - w_L^1) - c_1 \geq w_L^2 + p_L(w_H^2 - w_L^2)$$
$$w_L^1 + p_H(w_H^1 - w_L^1) - c_1 \geq w_L^2 + p_H(w_H^2 - w_L^2) - c_1$$

The IC constraints for type 1 are

$$
\begin{aligned}
w_L^2 + p_L(w_H^2 - w_L^2) &\geq w_L^2 + p_H(w_H^2 - w_L^2) - c_2 \\
w_L^2 + p_L(w_H^2 - w_L^2) &\geq w_L^1 + p_L(w_H^1 - w_L^1) \\
w_L^2 + p_L(w_H^2 - w_L^2) &\geq w_L^1 + p_H(w_H^1 - w_L^1) - c_2
\end{aligned}
$$

Since the principal does not wish type 2 to exert effort, they can set $w_L^2 = w_H^2 = w_2$. Since the principal wishes type 1 to exert high effort they can set $w_L^1 = 0$. The IC constraints thus reduce to

$$
\begin{aligned}
p_H w_H^1 - c_1 &\geq p_L w_H^1 \\
p_H w_H^1 - c_1 &\geq w_2
\end{aligned}
$$

The IC constraints for type 1 are

$$
\begin{aligned}
w_2 &\geq p_L w_H^1 \\
w_2 &\geq p_H w_H^1 - c_2
\end{aligned}
$$

These constraints are satisfied if $w_H^1 = c_1/\Delta(p)$ and $w_2 = p_L c_1/\Delta(p)$. Notice that the presence of type 2 agents means that it is more expensive to make type 1 agents choose H since this results in rents or type 2.

4. A principal employs an agent to work on a project. The worker chooses unobserved effort $e \in \{L, H\}$ at costs c_L and c_H, with $0 < c_L < c_H$. The project succeeds with probability p_H is $e = H$ and p_L if $e = L$, where $1 > p_H > p_L > 0$. The principal pays w_0 if the project fails and w_1 if it succeeds. The agent's utility is given by $u(w_s) - c_e$, where $s \in \{0, 1\}$. Utility $u(\cdot)$ is strictly increasing and strictly concave.The agent has reservation utility $\underline{U}$. The principal profit's profit is $x_s - w_s$, where x_1 is the output when the project succeeds, and x_0 is the output when the project fails, and $x_1 > x_0$.

(a) Suppose the principal chooses to implement $e = L$.

- (a_1) [5 marks] Write down the principal's maximisation problem.
- (a_2) [10 marks] What wages will the principal offer in the profit-maximising contract?

(b) [20 marks] Now suppose the principal chooses to implement $e = H$.

- (b_1) [10 marks] Write down the principal's maximisation problem.
- (b_2) [15 marks] What wages will the principal offer in the profit-maximising contract?
- (b_3) [10 marks] Show that in the profit-maximising contract derived in part b_2 it holds that $w_1 > w_0$.

SOLUTION 4.
a_1. The problem is

$$\max_{w_1, w_0} p_L(x_1 - w_1) + (1 - p_L)(x_0 - w_0)$$

subject to the IR constraint

$$p_L u(w_1) + (1 - p_L)u(w_0) - c_L \geq \underline{U}$$

and the IC constraint

$$p_L u(w_1) + (1 - p_L)u(w_0) - c_L \geq p_H u(w_1) + (1 - p_H)u(w_0) - c_H$$

a_2 Ignore the IC and write down the lagrangian. Taking the FOC with respect to w_0 and then with respect to w_1, we obtain

$$\frac{1}{u'(w_0)} = \frac{1}{u'(w_1)} = \lambda$$

where λ is the multiplier for the IR constraint. By strict concavity this implies that $w_1 = w_0 = w$. We can now use IR to obtain $u(w) = c_L + \underline{U}$. We now observe that since the wage is constant, IC holds.

b_1The problem is

$$\max_{w_1,w_0} p_H(x_1 - w_1) + (1 - p_H)(x_0 - w_0)$$

subject to the IR constraint

$$p_H u(w_1) + (1 - p_H)u(w_0) - c_H \geq \underline{U}$$

and the IC constraint

$$p_H u(w_1) + (1 - p_H)u(w_0) - c_H \geq p_L u(w_1) + (1 - p_L)u(w_0) - c_L$$

b_2 Write the lagrangian with λ the multiplier for IR and μ the multiplier for IC, take the FOC with respect to w_0 and w_1 and obtain

$$\frac{1}{u'(w_0)} = \lambda + \mu[1 - \frac{1 - p_L}{1 - p_H}]$$

and

$$\frac{1}{u'(w_1)} = \lambda + \mu[1 - \frac{p_L}{p_H}]$$

Now note that $\mu > 0$, for otherwise $w_0 = w_1$ and therefore IC will be violated.

b_3 Since $\mu > 0$ it is easy to verify that $w_1 > w_0$.

End of Section B

End of Paper