

Mid Term Ec992. 2010/2011

Answer ALL the questions. Two Hours.

Exercise 1.[25 Marks]

Consider the following auction. An object is auctioned off to N bidders. Bidder i 's valuation of the object is v_i . The auction rules are that each player submits a bid (a nonnegative number) in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object but pays the auctioneer the amount of the second-highest bid. If more than one bidder submits the highest bid, each gets the object with equal probability.

1. [15 marks] Show that submitting a bid of v_i with certainty is a weakly dominant strategy for bidder i ;
2. [10 marks] Argue that this is bidder i 's unique weakly dominant strategy.

Exercise 2.[25 marks]

Construct the set of rationalizable actions of each player in the two-player game depicted below.

$(1, 2)$	b_1	b_2	b_3	b_4
a_1	1, 8	1, 2	8, 1	3, 6
a_2	6, 3	1, 2	6, 3	4, 4
a_3	1, 1	11, 0	1, 1	3, 6
a_4	8, 1	1, 2	1, 8	1, -1

Exercise 3.[25 marks]

Consider the game represented in Figure 1.

1. [15 marks] Construct the unique Weak Perfect Bayesian Equilibrium of this game.
2. [10 marks] Find a Nash equilibrium which is not a Weak Perfect Bayesian Equilibrium.

Exercise 4.[25 marks] Provide a formal prove of the following statement.

A strategy profile σ is a Nash equilibrium of the extensive form game Σ_E if and only if there exists a system of beliefs μ such that:

1. the strategy profile σ is sequentially rational given belief system μ at all information sets H such that $\Pr(H|\sigma) > 0$;
2. The system of beliefs μ is derived from strategy profile σ through Bayes' rule whenever possible.