Mid Term Ec992. 2010/2011

Answer ALL the questions. Two Hours.

Exercise 1.[25 Marks]

Consider the following auction. An object is auctioned off to N bidders. Bidder i's valuation of the object is v_i. The auction rules are that each player submits a bid (a nonnegative number) in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object but pays the auctioneer the amount of the second-highest bid. If more that one bidder subits the highest bid, each gets the object with equal probability.

1. [15 marks] Show that submitting a bid of v_i with certainty is a weakly dominant strategy for bidder i;
2. [10 marks] Argue that this is bidder i's unique weakly dominant strategy.

Solution. Suppose not. Assume bidder i bids $b_i > v_i$. Then of some other bidder bids something larger than b_i, bidder i is just as well pf as of he would have bid v_i. If all other players bid lower than v_i, then bidder i obtains the object and pays the amount of the second highest bid. If the second highest bis is $b_j < v_i$, this results in the same payoff for player i i as if he bids v_i. However, suppose that the second highest bid of the other is $b_j > v_i$. Then, by bidding v_i he will not win the object and obtain a payoff of zero. Therefore, bidding $b_i > v_i$ is weakly dominated by bidding $b_i = v_i$.

Suppose bidder i bids $b_i < v_i$. Then if all other bidders bid something smaller than b_i, bidder i is just as well of as if he would have bid v_i. He will win the object and pay the second highest bid. If some other player bids higher than v_i, then bidder i does not win the object regardless of whether he bids b_i or v_i. However, suppose that nobody bids higher than v_i and the highest bid of the other plaeyrs is $b_j \in (b_i, v_i)$. Then by bidding b_i bidder i will not win the object, therefore getting payoff of 0. By bidding v_i he will win the object, pay $b_j < v_i$, and thus obtain positive profit. Therefore, bidding $b_i < v_i$ is weakly dominated by bidding v_i.

So, we have shown that $b_i - v_i$ is a unique weakly dominant strategy.

Exercise 2.[25 marks]

Construct the set of rationalizable actions of each player in the two-player game depicted below. TO MODIFY

$(1,2)$	b_1	b_2	b_3	b_4
a_1	$1,8$	$1,2$	$8,1$	$3,6$
a_2	$6,3$	$1,2$	$6,3$	$4,4$
a_3	$1,1$	$11,0$	$1,1$	$3,6$
a_4	$8,1$	$1,2$	$1,8$	$1,-1$

Solution

The set of rationalizable actions is: $\{a_1, a_2, a_4\}\{b_1, b_3, b_4\}$

Exercise 3.[25 marks]

Consider the game represented in Figure 1.

1. [15 marks] Construct the unique Weak Perfect Bayesian Equilibrium of this game.
2. [10 marks] Find a Nash equilibrium which is not a Weak Perfect Bayesian Equilibrium.

Solution

The unique WPBE is:

Strategy of firm E1: propose join venture, In if E2 declinesStrategy of firm E2: acceptStrategy of firm I: accomodateBeliefs: probability 1 to middle node in the tree

A Nash equilibrium which is not WPBE isStrategy of firm E1: Out, Out if E2 declinesStrategy of firm E2: declineStrategy of firm I: fight

Exercise 4.[25 marks] Provide a formal prove of the following statement.

A strategy profile σ is a Nash equilibrium of the extensive form game Σ_E if and only if there exists a system of beliefs μ such that:

1. the strategy profile σ is sequentially rational given belief system μ at all information sets H such that $\Pr(H|\sigma) > 0$;

2 The system of beliefs μ is derived from strategy profile σ through Bayes' rule whenever possible.

Solution

First, if σ is a NE then (1) and (2) must hold. If (1) weren't satisfied, then some player's information set is reached with psitive probability in which he is not playing a best response to his opponents' strategies, contradicting the fact that σ is a NE. If (2) were not satisfied then some player's information set is reached with positive probability in which his beliefs are not correct given his, and his opponents' strategies. Therefore, this cannot be an equilibrium.

Second, if (1) and (2) are satisfied, then σ is clearly a NE since at each information set which is reached with positive probability, sequential rationality implies that each player is playing a best response to his opponents' strategies, with correct beliefs at each such information set.