UNIVERSITY OF ESSEX
DEPARTMENT OF ECONOMICS

Advanced Microeconomics

(EC992-8-AU)

2011/2012

Midterm Exam - Solutions

Answer ALL the questions. Two hours.

1. Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them £100 in profits, but they must agree on how to split the £100. Bargaining works as follows: The two individuals each make a demand simultaneously. If their demands sum to more than £100, then they fail to agree and each gets nothing. If their demands sum to less than £100, they do the project, each gets his demand, and the rest goes to charity.

   (a) [10 Marks] What are each player's strictly dominated strategies?
   A: No player has strictly dominated strategies. Note that if player j demands $x_j > 100$*, player i's payoff is zero regardless of i's demand.*

   (b) [10 Marks] What are each player's weakly dominated strategies?
   A: Any demand $x_i > 100$ *is a weakly dominated strategy for individual i. The payoff to individual i from playing this strategy is 0 regardless of the demand* x_j *of individual j. Such a strategy is weakly dominated, for example, by demanding 10. If individual i demands 10, she obtains 10 if* $x_j \leq 90$ *and 0 if otherwise. A demand* $x_i = 0$ *is also weakly dominated for individual i. The payoff to individual i from demanding 0 is also 0 regardless of the demand of player j.*

   (c) [10 Marks] What are the pure strategy Nash equilibria of this game?
   A: Any profile of demands (x_1, x_2) *such that* $x_1 + x_2 = 100$ *and* $0 \leq x_1 \leq 100$ *(and, trivially,* $0 \leq x_2 \leq 100$*) constitutes a Nash equilibrium. Note that if individual i demands* $0 \leq x_i \leq 100$*, the best response of player j is to demand* $x_j = 100 - x_i$*. Any profile of demands* (x_1, x_2) *such that* $x_1 \geq 100$ *and* $x_2 \geq 100$ *also constitutes a Nash equilibrium, as in this case each individual obtains zero but cannot obtain more than 0 by changing her demand.*

2. Player 1, the "government," wishes to influence the choice of player 2. Player 2 chooses an action $a_2 \in A_2 = \{0, 1\}$ and receives a transfer $t \in T = \{0, 1\}$ from the government, which observes a_2. Player 2's objective is to maximize the expected value of his transfer, minus the cost of his action, which is 0 for $a_2 = 0$ and 1/2 for $a_2 = 1$. Player 1's objective is to minimize the sum $2(a_2 - 1)^2 + t$. Before player 2 chooses his action, the government can announce a transfer rule $t(a_2)$.

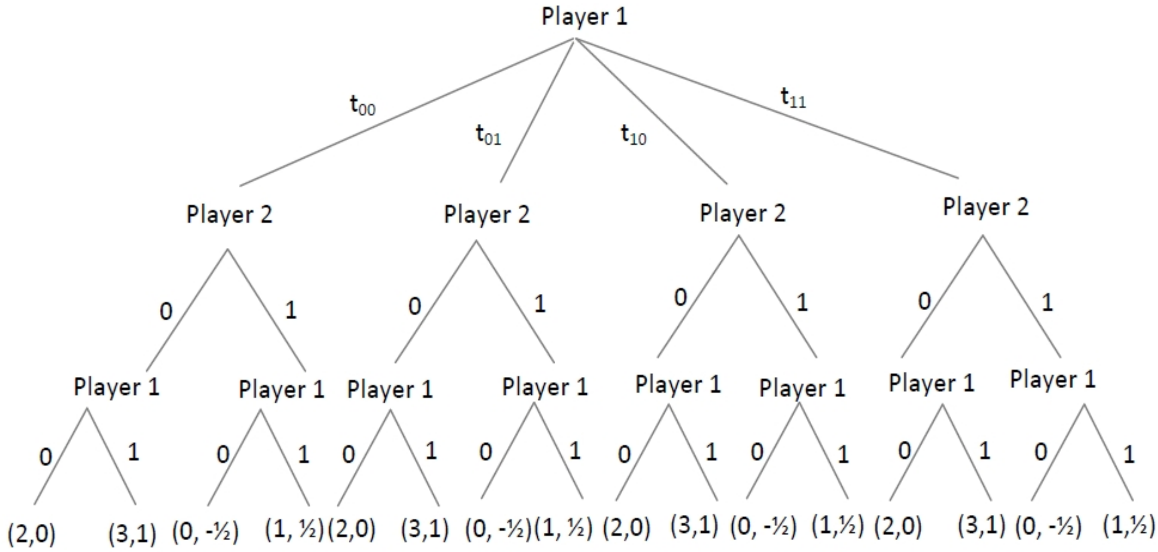
   (a) [7 Marks] Identify all the possible transfer rules $t(a_2)$.
   A: *There are four possible transfer rules:*

$$t_{00}(a_2) = \begin{cases} 0 \text{ if } a_2 = 0 \\ 0 \text{ if } a_2 = 1 \end{cases} \qquad t_{01}(a_2) = \begin{cases} 0 \text{ if } a_2 = 0 \\ 1 \text{ if } a_2 = 1 \end{cases}$$

$$t_{10}(a_2) = \begin{cases} 1 \text{ if } a_2 = 0 \\ 0 \text{ if } a_2 = 1 \end{cases} \qquad t_{11}(a_2) = \begin{cases} 1 \text{ if } a_2 = 0 \\ 1 \text{ if } a_2 = 1 \end{cases}$$

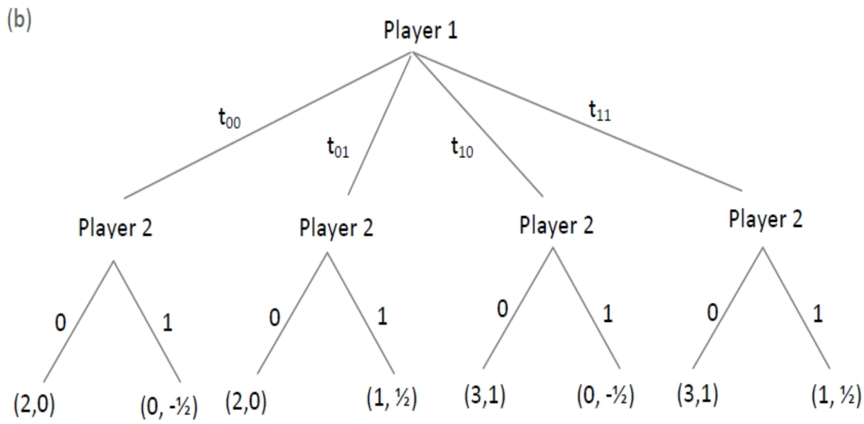
(b) [8 Marks] Draw the extensive form of the game for the case where the government's announcement is not binding and has no effect on payoffs.

A: *When the government's announcement is not binding, the government can choose a transfer after player 2's choice of action which does not satisfy the rule initially announced. Hence, the extensive form representation of the game is:*



(c) [8 Marks] Draw the extensive form of the game for the case where the government is constrained to implement the transfer rule it announced.

A: *In this case, the government is not free to choose the transfer after player 2's choice of action. The government's transfer has to respect the transfer rule initially announced. Hence, the extensive form representation of the game is:*



(d) [7 Marks] Characterize the SPNE of the two games.

A: *We can use backward induction to solve both games. Consider first the game without commitment by the government. Player 1 always chooses* 0 *at date 3. Player 2 always chooses* 0 *at date 2. At date 1, player 1 is indifferent between the 4 transfer rules. Hence, we have 4 SPNEs, one for each different choice of transfer rule by player 1 at date 1. Consider now the game with commitment by the government. There is one SPNE: player one choose* t_{01}*; and player 2 chooses 1 if player 1 chooses* t_{01} *and 0 if otherwise.*

3. Consider the following labor market model. There are many identical firms that can hire workers. Each produces the same output using an identical constant returns to scale technology in which labor is the only input. The firms are risk neutral, seek to maximise their expected profits, and act as price takers. Workers differ in the number of units of output they produce if hired by a firm, which is denoted by θ. Workers observe their productivity. Firms do not observe workers' productivity but know that θ is distributed uniformly on $[0, 2]$. A worker can decide whether to work or stay home. The utility to stay home of a worker of type θ is $r(\theta) = \alpha\theta$, where

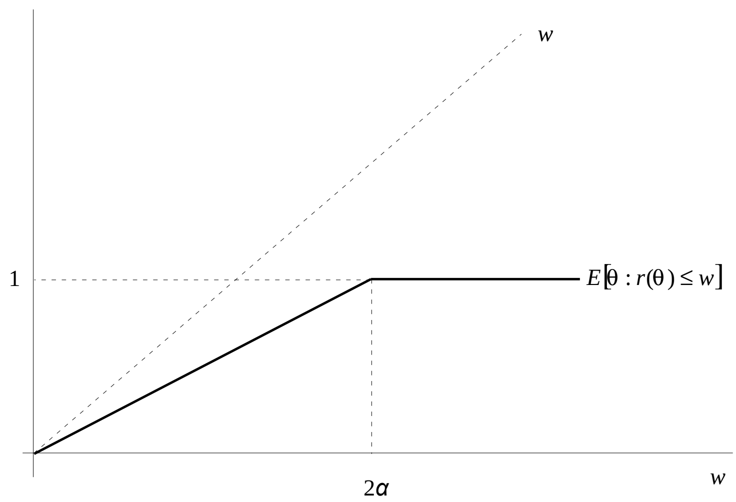
$0 < \alpha < 1$. In case of indifference between staying home and being employed, a worker chooses to be employed.

(a) [5 Marks] Show that $E[\theta \mid r(\theta) \leq w] = \frac{w}{2\alpha}$.
A: *Note that* $E[\theta \mid r(\theta) \leq w] = E[\theta \mid \alpha\theta \leq w] = E[\theta \mid \theta \leq w/\alpha]$. *Since* θ *is uniformly distributed on* $[0, 2]$, *we know that the density is* $1/2$, *which means that*

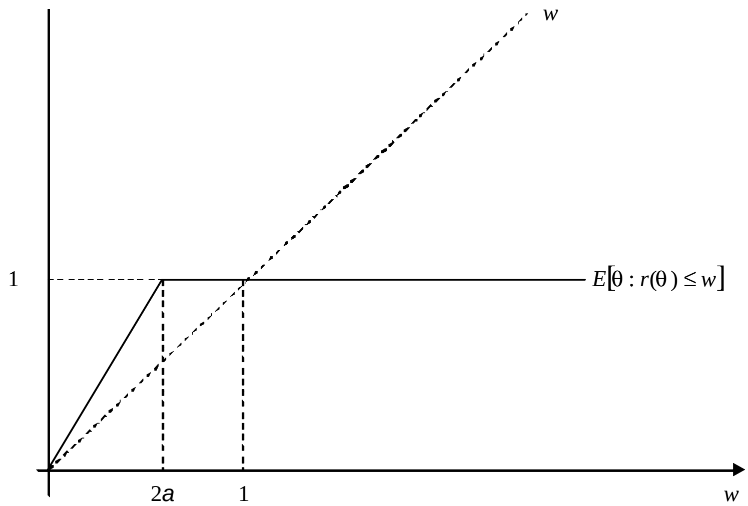
$$E[\theta \mid \theta \leq w/\alpha] = \int_0^{\frac{w}{\alpha}} \theta \frac{\frac{1}{2}}{\Pr[\theta \leq w/\alpha]} d\theta = \int_0^{\frac{w}{\alpha}} \theta \frac{1}{2} \frac{1}{\frac{1}{2}\frac{w}{\alpha}} d\theta = \int_0^{\frac{w}{\alpha}} \theta \frac{\alpha}{w} d\theta = \frac{w}{2\alpha}.$$

Obvioulsy this holds for $w/\alpha \leq 2$, *which is equivalent to* $w \leq 2\alpha$. *For* $w > 2\alpha$, $E[\theta \mid r(\theta) \leq w] = E[\theta] = 1$.

(b) [10 Marks] Suppose that $\alpha > \frac{1}{2}$. Find all the competitive equilibria.
A: *In a competitive equilibrium the wage satisfies* $w = E[\theta \mid r(\theta) \leq w]$. *Using the characterization of* $E[\theta \mid r(\theta) \leq w]$ *obtained in a), we obtain that there is only one solution to that equation which is* $w = 0$. *Hence there is only one competitive equilibrium in which* $w = 0$ *and only the workers of type* $\theta = 0$ *are employed. The following ficure illustrates the equilibrium.*



(c) [10 Marks] Suppose that $\alpha < \frac{1}{2}$. Find all the competitive equilibria.
A: *Using again the characterization of* $E[\theta \mid r(\theta) \leq w]$ *obatined in a) we obtain that both* $w = 0$ *and* $w = 1$ *solve* $w = E[\theta \mid r(\theta) \leq w]$. *Hence, there are two competitive equilibria: (i) one in which* $w = 0$ *and only workers of type* $\theta = 0$ *are employed; and (ii) one in which* $w = 1$ *and all workers are employed. The following figure illustrates the two equilibria.*



4. [15 Marks] A worker knows her talent $\theta \in \{\theta_L, \theta_H\}$, where $\theta_H > \theta_L > 0$. The employer does not know the talent of the worker but believes that $\lambda = \Pr(\theta = \theta_H) \in (0, 1)$. In the first stage the worker decides a level of education $e > 0$. The employer observes the level of education chosen by the worker and offers the worker a wage which equals the worker's expected productivity. Finally, the worker accepts or rejects the offer.

   Education does not affect the productivity of the worker, which is identical to her talent. The cost of obtaining education level e for a θ type worker is given by the twice continuously differentiable function $c(e, \theta)$, with $c(0, \theta) = 0$ and $c_e(e, \theta) > 0$, $c_{ee}(e, \theta) > 0$, $c_\theta(e, \theta) < 0$ and $c_{e\theta}(e, \theta) < 0$, where subscripts denote partial derivatives. The utility of a type θ worker who chooses education level e, receives wage w and accepts the offer is $u(w; e \mid \theta) = w - c(e, \theta)$. It is also assumed that a worker of talent θ can earn $r(\theta)$ by working home and that $r(\theta_L) = r(\theta_H) = 0$.

   Show that in any separating equilibrium of this game (if such an equilibrium exists) the worker of type θ_L chooses education level $e = 0$.

   A: *Suppose that there is a separating equilibrium in which the worker of type θ_L chooses education level $e_L > 0$ and the worker of type θ_H chooses education level $e_H \neq e_L$. In this separating equilibrium the employer's beliefs about the type of the worker must satisfy: $\mu(e_L) = 0$ and $\mu(e_H) = 1$, where $u(e)$ denotes the employer's belief that the work is of type θ_H given education level e. Hence, the wage of the worker of type θ_L must be $\mu(e_L)\theta_H + (1 - \mu(e_L))\theta_L = \theta_L$, and his utility $\theta_L - c(e_L, \theta_L)$. Suppose now that the worker deviates and chooses $e'_L = 0$. His wage will be $w'_L = \mu(0)\theta_H + (1 - \mu(0))\theta_L \geq \theta_L$ and his utility will be $w'_L - c(0, \theta_L)$, which is strictly greater than $\theta_L - c(e_L, \theta_L)$ because $c(e, \theta_L)$* increases with *e. This contradicts the initial supposition of equilibrium, and completes the proof.*